



MCS CONTENT STANDARDS FOR 3rd GRADE MATHEMATICS

Fluency Expectations or Examples of Culminating Standards

- 3.OA.7: ***Fluently*** multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows that $40 \div 5 = 8$) or properties of operations. Know from memory all products of two one-digit numbers; **and fully understand the concept when a remainder does not exist under division.**
- 3.NBT.2: ***Fluently*** add and subtract within 1000 (**including subtracting across zeros**) using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **Include problems with whole dollar amounts.**

The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards:

Significant Changes (ex: change in expectations, new Standards, or removed Standards)

3.OA.7

3.NBT.2

Slight Changes (slight change or clarification in wording)

3.OA.4 3.NF.3a

3.OA.6 3.G.1

3.OA.8 3.MD.7b


Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades K-5 Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word ***fluently*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

2016 Mississippi College- and Career-Readiness Standards for Mathematics, p. 19

| Operations and Algebraic Thinking | |
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| Cluster | |
| Represent and solve problems involving multiplication and division. | |
| Vocabulary: factors, products, quotients, multiplication, division, equal groups, group size, arrays, equations, unknowns | |
| Standard | Clarifications |
| <p>3.OA.1</p> <p>Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each</p> <p><i>For example, describe a context in which a total number of objects can be expressed as 5×7.</i></p> | <p>The foundation for multiplication is introduced in Grade 2:</p> <ul style="list-style-type: none"> • 2.OA.4 – Students work with rectangular arrays (limited to a max of 5 rows and/or 5 columns) and use <u>repeated addition</u> to find the total number of objects and to write an equation, representing the sum as a total of equal addends. • 2.NBT.2 – Students skip count by 5s and 10s (and 100s). <p>In Grade 3, students extend this work to use multiplication as a form of repeated addition. Students learn that the symbol “\times” means “groups of” and problems such as 5×7 can be translated as “5 groups of 7.” Thinking in terms of multiplication can be challenging for students, as it pushes them to think about <i>groups</i> of items, rather than <i>individual</i> items that they can count. Initially, students may still skip-count to find small products, but they will be expected not to depend on this strategy by the end of Grade 3.</p> <p>The game “Circles and Stars” can be used to help students build on their prior work with repeated addition and move forward into a deeper understanding of multiplication.</p> <p>Table 2 (at the end of this document) describes scenarios that students should explore in multiplication. Below is an example of an “Equal Groups, Unknown Product” problem that also fits this Standard:</p> <p><u>Example:</u> Jody bought 3 bags of cookies at the bake sale. Each bag had 4 cookies in it. How many cookies did Jody buy? Draw a picture to model and solve the problem, and explain how you used your picture to answer the question.</p> <div style="text-align: right;">  </div> <p><u>Student:</u> Jody bought 3 bags, so I drew 3 circles to show the bags. Then I put four circles in each bag to show 4 cookies in each one. I put three fours together in my head: four, eight, twelve. So, she bought 12 cookies.</p> <p><u>Teacher:</u> What number sentence best describes how you thought about the problem?</p> <p><u>Student:</u> I made 3 groups of 4 cookies, so I would say $3 \times 4 = 12$.</p> <p><u>Teacher:</u> What about $4 \times 3 = 12$? Could you use that number sentence, too?</p> <p><u>Student:</u> No, $4 \times 3 = 12$ tells you something different. 4×3 means “4 groups of 3.” She didn’t buy 4 bags with 3 cookies. You would still get 12 cookies, but your picture wouldn’t fit the story.</p> |

3.OA.2

Interpret whole-number quotients of whole numbers, e.g. interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

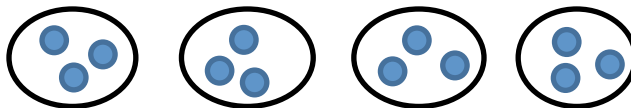
This Standard focuses on two distinct models of division: “partitioning” (or “partitive”) models and “measurement” (or “repeated subtraction”) models. These problems are also described in Table 2.

In a partitioning model of division (“Group Size Unknown”), the total number of objects is known and the total number of (desired) groups is known. The unknown (and usual focus of the question) is how much or how many objects should be in each group so that each group has the same amount.

Example of Partitioning Story Problem:

Zoe is having a tea party for her dolls. She has 4 dolls and 12 cookies. How many cookies should she give each doll so that they each get a fair share?

Children typically model partitioning by creating the number of groups (ex: drawing four circles, boxes, or smiley faces to represent four people) and then passing out (“partitioning”) one or more objects at a time until the groups have the same amount. The answer to the question is then the number of objects in each group.

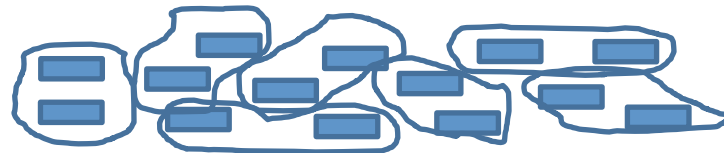


The number sentence should reflect the actions taken to model and solve the problem. For “Zoe’s Tea Party,” this would be $12 \div 4 = 3$. (12 cookies split into 4 equal groups puts 3 cookies in each group.)

In a repeated subtraction model of division (“Number of Groups Unknown”), the total number of objects is known and the total number of objects in each (desired) group is known. The unknown (and usual focus of the question) is how many equal-sized groups will be made.

Example of a Repeated Subtraction Story Problem:

Josh is making favors for his birthday party. He has 14 mini candy bars, and he wants to put 2 bars in each bag. How many bags does Josh need?



Children typically model repeated subtraction by making a representation of the total number of objects and then circling or scooping away (“repeatedly subtracting”) the number of objects in each group. The answer to the question is then how many groups they made.

The number sentence should reflect the actions taken to model and solve the problem. For “Josh’s Party,” this would be $14 \div 2 = 7$. (14 split into groups with 2 in each group makes 7 groups.)

(continued on next page)

| <p>3.OA.2 (<i>cont'd</i>)</p> <p>Interpret whole-number quotients of whole numbers, e.g. interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.</p> | <p>TEACHER NOTE: It can be difficult for adults not to project their own knowledge of multiplication facts when modeling these story problems with children. Adults are often tempted to say, “I made 4 groups and put 3 in each group,” or “I drew 14 candy bars and circles 7 groups of 2.” Adults <i>already know</i> the answer. But children <u>are using the models to discover and learn these relationships</u>. Thus, it is vital for teachers to also <u>use the model</u> to “discover” the answer in these scenarios; not to “make” the answer without really going through the process.</p> <p>TEACHER NOTE: The language that teachers use when discussing division is also critical. Teachers may be tempted to say, “Always divide the bigger number by the smaller number”; but this is not always true. When students begin working with fractions and decimals, that “rule” falls apart, but it is very difficult to help students “unlearn” that phrase. Teachers should not use this language when discussing division.</p> | | | | | | | | | | |
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| <p>3.OA.3</p> <p>Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings, and equations with a symbol for the unknown number to represent the problem.¹</p> <p>¹ See Glossary, Table 2</p> | <p>Table 2 (included at the end of this document) gives examples of a variety of problem-solving contexts in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures.</p> <p>This Standard is clearly connected to 3.OA.1 and 3.OA.2. (See examples provided earlier.) Pictures/models are <u>key</u> within this Standard as means of representing and solving problems and should not be neglected.</p> <p><u>Example</u> (“Arrays, Unknown Product”): Tracy has a garden. She has 3 rows of 6 sunflowers in her garden. How many sunflowers does Tracy have? Draw a picture to model the problem. Write an equation that describes what you are trying to find.</p> <p><u>Student:</u> I used x’s to be the sunflowers because that was easier for me to draw. × × × × × × I drew 3 rows of 6 x’s to show the garden. I want to know how many are in × × × × × × 3 rows of 6, so my equation would be $3 \times 6 = \square$. × × × × × ×</p> <p>Then I counted up my x’s and got 18. Tracy has 18 sunflowers.</p> | | | | | | | | | | |
| <p>3.OA.4</p> <p>Determine the unknown whole number in a multiplication or division equation relating three whole numbers, with factors 0-10.</p> <p><i>For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$</i></p> <p>(continued on next page)</p> | <p>In this Standard, students build on their understanding of multiplication/division as describing <u>relationships</u> between numbers. Students are not expected to use pictures here, but the emphasis should still be on understanding what the numbers and symbols are describing.</p> <table border="1" data-bbox="701 1243 1885 1451"> <thead> <tr> <th>Equation</th><th>Reasonable Interpretation</th></tr> </thead> <tbody> <tr> <td>$8 \times ? = 48$</td><td>“8 groups of something equals 48.” or “8 groups of what equals 48?”</td></tr> <tr> <td>$5 = \square \div 3$</td><td>“5 equals some number split into 3 groups.” or “What amount split into 3 groups would have 5 things in each group?”</td></tr> <tr> <td>$6 \times 6 = ?$</td><td>“6 groups of 6 is how many?”</td></tr> <tr> <td>$\square = 2 \times 7$</td><td>“How many is 2 groups of 7?”</td></tr> </tbody> </table> | Equation | Reasonable Interpretation | $8 \times ? = 48$ | “8 groups of something equals 48.” or “8 groups of what equals 48?” | $5 = \square \div 3$ | “5 equals some number split into 3 groups.” or “What amount split into 3 groups would have 5 things in each group?” | $6 \times 6 = ?$ | “6 groups of 6 is how many?” | $\square = 2 \times 7$ | “How many is 2 groups of 7?” |
| Equation | Reasonable Interpretation | | | | | | | | | | |
| $8 \times ? = 48$ | “8 groups of something equals 48.” or “8 groups of what equals 48?” | | | | | | | | | | |
| $5 = \square \div 3$ | “5 equals some number split into 3 groups.” or “What amount split into 3 groups would have 5 things in each group?” | | | | | | | | | | |
| $6 \times 6 = ?$ | “6 groups of 6 is how many?” | | | | | | | | | | |
| $\square = 2 \times 7$ | “How many is 2 groups of 7?” | | | | | | | | | | |

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| <p>3.OA.4 (cont'd)</p> <p>Determine the unknown whole number in a multiplication or division equation relating three whole numbers, with factors 0-10</p> | <p>TEACHER NOTE: The most common misinterpretation of the equal sign is that it means, “The answer comes next,” because most textbooks emphasize equations of that form (ex: $7 \times 5 = \square$ or $36 \div 4 = \square$)</p> <p>Working with equations in which the unknown is in a different position (ex: $\square = 15 \div 3$ or $18 = \square \times 2$) provides important opportunities to address this misconception in class discussions.</p> |
| <p>Cluster</p> | |
| <p>Understand properties of multiplication and the relationship between multiplication and division.</p> | |
| <p>Vocabulary: multiply, divide, factor, product, quotient, unknown, strategies, properties (rules about how numbers work)</p> | |
| <p>3.OA.5</p> <p>Apply properties of operations as strategies to multiply and divide.²</p> <p><i>Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</i></p> <p>² Students need not use formal terms for these properties.</p> <p>(continued on next page)</p> | <p>The purpose of this Standard is to provide students with opportunities to explore number relationships and see the value in properties of operations as problem-solving strategies.</p> <p>The footnote in the Standard says that students “need not use formal terms for these properties.” So, it is not the expectation that students verbalize The Commutative Property of Multiplication as “$a \times b = b \times a$.” However, when students notice relationships between numbers (ex: 3 groups of 4 and 4 groups of 3 both have a total of 12 objects), they can be introduced to “the math name” for that relationship – The Commutative Property of Multiplication. This name can even be added to a word wall. <u>Students should not be taught incorrect terminology</u> (ex: “The Switcharoo Property”) in place of accurate terminology.</p> <p>The Commutative Property is important because not all operations have this property. For example, there is no Commutative Property of Subtraction or Commutative Property of Division. It can be powerful for children to explore modeling these relationships to see that while 3×4 and 4×3 produce the same amount, $10 \div 5$ (ex: 10 cookies split among 5 friends) <i>does</i> not have the same result as $5 \div 10$ (ex: 5 cookies split among 10 friends.)</p> <p>In Grades 1 and 2, students had experience in looking for “friendly numbers” to use in addition/subtraction. In Grade 3, they build on that idea to look for friendly numbers to use in multiplication and division. Doing so promotes number sense and mathematical reasoning, as well as mental math strategies.</p> <p><u>Example of using friendly numbers in addition:</u></p> $\begin{array}{c} (3) + 9 + (7) = 10 + 9 = 19 \\ \quad \swarrow \quad \searrow \\ \quad \quad 10 \end{array}$ <p><u>Example of using friendly numbers in multiplication:</u></p> $\begin{array}{c} 8 \times (2) \times (5) = 8 \times 10 = 80 \\ \quad \quad \swarrow \quad \searrow \\ \quad \quad \quad 10 \end{array}$ <p>This, of course, is an example of the Associative Property of Multiplication: Whether we “group” the factors to multiply them as $(8 \times 2) \times 5 = 16 \times 5 = 80$ or as $8 \times (2 \times 5) = 8 \times 10 = 80$, we will still get the same product.</p> |

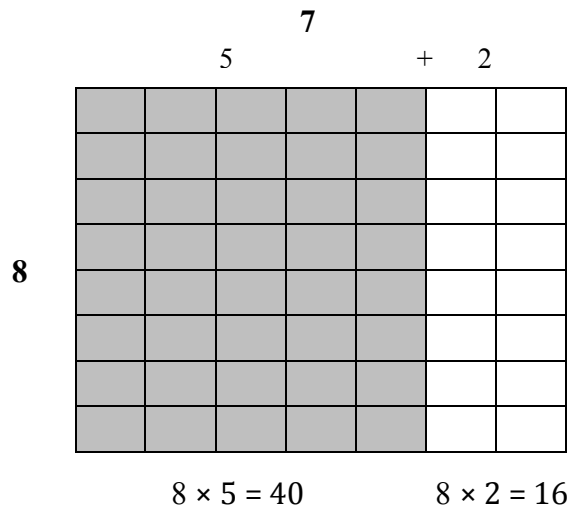
3.OA.5 (cont'd)

Apply properties of operations as strategies to multiply and divide.²

Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

² Students need not use formal terms for these properties.

Using an array model to represent multiplication can help students visualize and see the usefulness of the Distributive Property of Multiplication over Addition. For example, suppose we drew an array to represent “ $8 \times 7 = ?$ ” By splitting the array into smaller arrays that let us use more familiar mental math facts (8×5 and 8×2), students are using The Distributive Property of Multiplication in a meaningful way:



$40 + 16 = 56$. So, $8 \times 7 = 56$. This array model shows that $8 \times 7 = 8 \times (5 + 2) = 40 + 16 = 56$.

A powerful relationship that is often neglected is The Distributive Property of Multiplication over Subtraction. For example, many students struggle to remember their “9” multiplication facts. However, given the opportunity, students can make the connection that 9 is equal to $10 - 1$, which can be used in mental math:

$$\begin{aligned} 9 \times 8 \\ &= (10 - 1) \times 8 \\ &= 80 - 8 = 72 \end{aligned}$$

Properties such as the Zero Property of Multiplication and the Identity Property of Multiplication can be linked back other multiplication strategies/reasoning in order make sense of these properties and give them context.

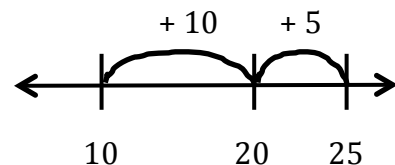
Examples: $7 \times 0 = 0$ “7 groups of 0 is just 0.” (3.OA.1)

$\square \times 0 = 0$ “Anything times 0 equals 0.” (The Zero Property of Multiplication)

$1 \times 9 = 9$ “1 group of 9 is just 9.” (3.OA.1)

$1 \times \square = \square$ “1 times any number is that same number.” (The Identity Property of Multiplication)

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| <p>3.OA.6 Understand division as an unknown-factor problem, where a remainder does not exist.</p> <p><i>For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.</i></p> | <p>Multiplication and division are inverse operations. If multiplication can be thought of as repeatedly adding equal groups, then division can be thought of as taking an amount and splitting it up into equal groups. (There are other ways to think about multiplication and division; this is just an example.)</p> <p>Fact families help students see the relationships between the multiplication and division so that they can make sense of and recognize “facts” with meaningful references, rather than memorizing facts separately.</p> <p><u>Example:</u> What is the multiplication/division fact family for 3, 5, and 15?</p> $3 \times 5 = 15$ $5 \times 3 = 15$ $15 \div 3 = 5$ $15 \div 5 = 3$ <p><u>Teacher:</u> How do you remember this fact family?</p> <p><u>Student:</u> Well, I know 3×5 is 15. That’s 3 groups of 5 or like, “5, 10, 15.” And I know from playing Circles and Stars that 3×5 and 5×3 give you the same amount. With division, you kind of think about it backwards. If 3 groups of 5 make 15, then 15 split into 3 groups is 5 in each group. I can see it in my head. And then 15 split into 5 groups puts 3 in each group. They’re all related, it just depends on what you’re doing.</p> |
| Cluster | |
| Multiply and divide within 100. | |
| Vocabulary: multiply, divide, factor, product, quotient, unknown, strategies, | |
| <p>3.OA.7 <i>Fluently</i> multiply and divide within 100 using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows that $40 \div 5 = 8$) or properties of operations. Know from memory all products of two one-digit numbers; and fully understand the concept when a remainder does not exist under division.</p> <p><i>(continued on next page)</i></p> | <p>“The word <i>fluent</i> is used in the Standards to mean ‘fast and accurate.’ Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., ‘adding 0 yields the same number’), and knowing some answers <i>from the use of strategies</i>. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that <u>fluency will be a mixture of these kinds of thinking</u> which may differ across students.” (<i>Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking (Draft)</i>, May 2011, p. 18)</p> <p>The goal of this Standard is <u>not</u> to have students memorize multiplication facts. “Knowing from memory” is the desired result of yearlong experiences in working with pictures, models, word problems, and other multiplicative contexts so that students internalize these relationships over time.</p> <p>There has been increasing research within the past several years describing the negative impact that timed tests and drills have on students. The overall consensus is that timed tests and flash card drills are not effective means to help students learn “facts” with long-term success. Students who are strong memorizers may have success with these assessments, but that does not mean that they <u>understand</u> the number relationships.</p> |

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| <p>3.OA.7 (cont'd)</p> <p>Fluently multiply and divide within 100 using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows that $40 \div 5 = 8$) or properties of operations. Know from memory all products of two one-digit numbers; and fully understand the concept when a remainder does not exist under division</p> | <p>For more information on the negative impact of timed tests and alternative assessment strategies that promote number sense and long-term success, see</p> <ul style="list-style-type: none">• Boaler, J. (2014, April). Research suggests that timed tests cause math anxiety. <i>Teaching Children Mathematics</i>, 20(8), 469-474.• Kling, G., & Bay-Williams, J. M. (2014, April). Assessing basic fact fluency. <i>Teaching Children Mathematics</i>, 20(8), 488-497. <p>Other helpful strategies for learning multiplication and division with understanding can be found in the examples provided for 3.OA.5 and 3.OA.6.</p> <p>This Standard does include multiplying one digit and two digit numbers whose product is less than 100.</p> | | | | | | | | | | |
| <p>Cluster</p> <p>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</p> <p>Vocabulary: multiply, divide, factor, product, quotient, subtract, add, addend, sum, different, equation, expression, unknown, strategies, reasonableness, mental math, estimation, rounding, patterns, properties (rules about how numbers work)</p> | | | | | | | | | | | |
| <p>3.OA.8</p> <p>Solve two-step (two operational steps) word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.³ Include problems with whole dollar amounts.</p> <p>³ This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</p> <p>(continued on next page)</p> | <p>This standard refers to two-step word problems using the four operations. Adding and subtracting numbers should include numbers within 1000 (3.NBT.2), and multiplying and dividing numbers should include single-digit factors and products less than 100 (3.OA.7).</p> <p><u>Example:</u> Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal?</p> <p><u>Student:</u> I used tally marks to figure out how far Mike ran in 5 days:</p> <table><tr><td><u>Day 1</u></td><td><u>Day 2</u></td><td><u>Day 3</u></td><td><u>Day 4</u></td><td><u>Day 5</u></td></tr><tr><td> </td><td> </td><td> </td><td> </td><td> </td></tr></table> <p>So, after 5 days, he'd run 10 miles. He wants to go 25 miles. So then I pictured jumping up from 10 to 25 on the number line in my head: 10 more would be 20 and then 5 more would be 25. 10 and 5 is 15, so he needs to run 15 more miles.</p>  <p><u>Teacher:</u> How would you write an equation to describe how you thought about the problem?</p> <p><u>Student:</u> Well, I thought about 5 groups of 2 first. That's 5×2. And then I wanted to figure out how much more I needed to equal 25. So the missing amount is how much I needed to add to 5 groups of 2 to get 25. So, I would write $5 \times 2 + ? = 25$.</p> | <u>Day 1</u> | <u>Day 2</u> | <u>Day 3</u> | <u>Day 4</u> | <u>Day 5</u> | | | | | |
| <u>Day 1</u> | <u>Day 2</u> | <u>Day 3</u> | <u>Day 4</u> | <u>Day 5</u> | | | | | | | |
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|---|--|
| <p>3.OA.8 (cont'd)</p> <p>Solve two-step (two operational steps) word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.³ Include problems with whole dollar amounts.</p> <p>³ This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</p> | <p>Students should be able to use number sense and estimation strategies to predict “about how much” an answer should be <u>before solving a problem</u>, not just to check the results of their work. By doing so, students can recognize mistakes in calculations along the way and can strengthen their number sense and reasoning skills.</p> <p><u>Examples of students’ rounding and estimation strategies in context:</u></p> <p>On a vacation, your family travels 267 miles on the first day, 194 miles the second day, and 34 miles the third day. How many total miles did they travel?</p> <p><u>Student 1:</u> I looked at 267 and 34 and thought, “That would be about 300. And then 197 is almost 200. So, the answer should be somewhere around 500.” When I worked it out, I got 495 miles, and that makes sense.</p> <p><u>Student 2:</u> I looked at the numbers in the problem and thought, “Okay, about 300, about 200, and about 30. So, my answer should be around 530.” When I worked it out, I got 801 miles, and I was like, “Whoa! What?” So, I went back and looked, and I’d written 34 down as 340 accidentally. I guess I was in a hurry. But then I fixed it and added them up again and got 495 miles, which is closer to what I thought.</p> <p><u>Student 3:</u> I rounded 267 to 300, 194 to 200, and 34 to 30. Then I put those together and got 530. My estimate is “less than 530” ‘cause I rounded up the numbers in the problem kind of a lot. When I added the real numbers from the story, I got 495 miles, which fits what I thought it’d be.</p> <p>The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a reasonable range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.</p> |
| <p>3.OA.9</p> <p>Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.</p> <p><i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i></p> <p>(continued on next page)</p> | <p><i>TEACHER NOTE:</i> An even number is an amount that can be made of two equal parts with no leftovers. An odd number is one that is not even or cannot be made of two equal parts. The number endings of 0, 2, 4, 6, and 8 are only an interesting and useful pattern or observation <i>and should not be used as the definition of an even number.</i> (Van de Walle & Lovin, 2006, p. 292)</p> <p>Students will likely have experience with a hundreds chart (addition table) from Grades 1 and 2. Discussions about “what is the same” and “what is different” between an addition table and a multiplication table can help students make sense of these representations and use them well.</p> <p><u>Examples of Patterns that Students May Notice and Should Investigate:</u></p> <ul style="list-style-type: none"> • The sum of two even numbers is even. • The sum of two odd numbers is even. • The sum of an even number and an odd number is odd. • The multiples of 4, 6, 8, and 10 are all even (because they can all be decomposed into two equal groups). <p>(continued on next page)</p> |

3.OA.9 (cont'd)

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends

Examples of Patterns that Students May Notice and Should Investigate (cont'd):

- The doubles in an addition table (2 adds the same) fall on a diagonal while the doubles in a multiplication table (multiples of 2) fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to The Commutative Property.
- All of the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Students may also investigate different representations of a 3-digit number and look for patterns in the number of ways you could represent the numbers of hundreds, tens, and ones.

Example:

I used the Base 10 Blocks to represent the number 243. I wrote down different ways I used the blocks in the table below.

| 243 | | |
|-------|-------|-------|
| flats | longs | units |
| 2 | 4 | 3 |
| 1 | 14 | 3 |
| 0 | 24 | 3 |
| | | |
| 2 | 3 | 13 |
| 2 | 2 | 23 |
| 2 | 1 | 33 |
| 2 | 0 | 43 |
| 2 | 3 | 13 |

Example cont'd: Do you notice any patterns in the number of flats, longs, and units that I used? Why do you think those patterns are happening?

Potential Responses Include:

- * Every time the number of flats decreases by 1, the number of longs increases by 10. That's because it takes 10 longs to make a flat, so you must have swapped them out.
- * Every time the number of longs decreases by 1, the number of units increases by 10. That's because it takes 10 units to make a long, so you must have swapped them out.
- * The number of units is always an odd number because 3 is in the ones place, and it's an odd number.
- * The biggest number in the flats is 2. If you used 3 flats, you'd have 3 hundreds. That's too much for 243.

Number and Operations in Base 10

Cluster

Use place value understanding and properties of operations to perform multi-digit arithmetic.⁴

⁴ A range of algorithms may be used.

Vocabulary: place value, round, addition, add, addend, sum, subtraction, subtract, difference, strategies

3.NBT.1

Use place value understanding to round whole numbers to the nearest 10 or 100.

This Standard refers to place value understanding, which extends beyond an algorithm or memorized procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example:

Jacob and Dylan used rounding to estimate the sum of 171 and 82. Jacob rounded the numbers to the nearest ten and used $170 + 80$ to estimate his answer. Dylan rounded the numbers to the nearest 100 and used 200 and 100 to estimate his answer.

What is the actual sum of 171 and 82? How did the boys' rounding methods affect their estimates?

3.NBT.2

Fluently add and subtract within 1000 (including subtracting across zeros) using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. Include problems with whole dollar amounts.

The Standards intentionally scaffold addition and subtraction strategies across Grades 1, 2, and 3. The goal is to help students move through developmentally appropriate progression of concrete → pictorial → symbolic stages of understanding, working with larger and larger numbers. Teachers should not skip or hurry through picture and/or modeling strategies when a Standard specifically calls for them.

| Grade 1 | Grade 2 | Grade 3 |
|--|--|---|
| Add within 100 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (1.NBT.4). | <i>Fluently</i> add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (2.NBT.5). | |
| | Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method (2.NBT.7). | <i>Fluently</i> add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (3.NBT.2). |

(continued on next page)

| | |
|--|--|
| <p>3.NBT.2 (<i>cont'd</i>)</p> <p><i>Fluently</i> add and subtract within 1000 (including subtracting across zeros) using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. Include problems with whole dollar amounts.</p> <p>(continued on next page)</p> | <p>Students are not expected to be fluent with the standard algorithms for addition and subtraction until the end of Grade 4 (4.NBT.4) However, students can and should have experience in using the traditional algorithm to add and subtract, as it is often efficient.</p> <p>Students should be able to explain their thinking (such as why they chose a particular model or method to solve the problem), and use number sense or estimation to make sure that their answer is reasonable.</p> <p>The word “algorithm” refers to a procedure or a series of steps that when followed will produce a correct solution. Students should be able to explain how they used the traditional algorithm based on understanding of place value and number. “Explanations” such as “I followed the steps.” or “More on the floor? Go next door!” are not mathematical explanations and do not demonstrate deep understanding.</p> <p>Historically when describing the steps of the standard algorithm, phrases such as, “You can’t take away a bigger number from a smaller number,” are used. <u>This statement should not be used in the classroom because it is inaccurate:</u></p> <p>(1) You <u>can</u> subtract a larger amount from a smaller amount. The result is what we call a negative number, often representing the absence of an amount. It is very difficult to get students to “unlearn” this statement when they begin exploring integers (positive and negative numbers) in the sixth grade. It would be better, both for the students’ long-term understanding and for consistency of instruction, if students were not presented with such explanations in elementary mathematics.</p> <p>(2) The phrase itself demonstrates an inaccurate interpretation of the mathematics at hand. Consider the problem $423 - 156 = \underline{\quad}$. One might hear, “You can’t take away 6 from 3; you can’t take away a bigger number from a smaller number!” as the first step in solving this problem. But this problem isn’t asking us to “remove 6 from 3”; it’s asking us to “remove 156 from 423.” <u>We’re not trying to take away a bigger number from a smaller number at all.</u> Language such as this strips away the relationship of the digits from the overall value of the number and should be avoided.</p> <p>“The word <i>fluent</i> is used in the Standards to mean ‘fast and accurate.’ Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., ‘adding 0 yields the same number’), and knowing some answers <i>from the use of strategies</i>. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that <u>fluency will be a mixture of these kinds of thinking which may differ across students.</u>” (<i>Progressions for the CCSSM (Draft): K, Cardinality; K-5, Operations and Algebraic Thinking, May 2011, p. 18</i>)</p> |
|--|--|

3.NBT.2 (cont'd)

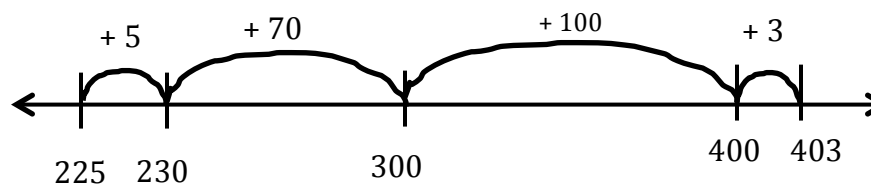
Fluently add and subtract within 1000 (including subtracting across zeros) using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. Include problems with whole dollar amounts

Through intentional selection of tasks and discussions facilitated by the teacher, students should begin to recognize when a certain method is particularly efficient (or inefficient) for solving a problem.

Example:

There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1: I used an empty number line model.



I started at 225, which was the number of fifth graders. I need to add 178. First I jumped 5 to get to 230 because it's easier for me to work with round numbers. Then I jumped 70 to get to 300. Then I jumped 100 for the 100 in 178. Now I'm at 400. I need to add 3 more for the 8 in the ones place because I added 5 when I first started. Then I landed at 403. I added up how far I jumped to check, and I'd jumped 178, which is the number of fourth graders. So, that's right. And so there are 403 total students.

Student 2:

$$\begin{array}{r} 178 \\ + 225 \\ \hline 300 \\ 90 \\ \hline 13 \\ 403 \end{array}$$

403 students

"Partial Sums" Algorithm

Student 3:

$$\begin{array}{r} 178 \\ + 225 \\ \hline 103 \end{array} \begin{array}{r} 75 \\ \hline \end{array} = 103 + 300 = 403$$

Student 4:

$$\begin{array}{r} 178 \\ + 225 \\ \hline 2 \end{array} \begin{array}{r} 223 \\ \hline \end{array} = 180 + 223$$

$$100 + 200 = 300$$

$$80 + 20 = 100$$

$$300, 400, 403$$

403 students

| | |
|--|--|
| <p>3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.</p> | <p><u>Students should not rely on tricks and superficial procedures such as “just add zeroes” that do not reflect mathematical understanding.</u> Technically, to “add zero” mathematically translates to “+ 0.” When someone says, “To do 6×20, I do 6×2 and then just add a 0,” what they have actually described in terms of mathematics is “$6 \times 2 = 12$. $12 + 0 = 12$,” which is clearly not the answer to 6×20.</p> <p>The Base 10 Blocks can be helpful for discussing multiplication with multiples of 10, focusing on place value and meaning. For example, for the problem 4×50, students could model 4 groups of 5 longs from the Base 10 blocks to show “4 groups of 50.” Using the Base 10 blocks and discussion questions, teachers might help students make the connection that “If 4 groups of 5 is 20, I can think of 4 groups of 50 as 4 groups of 5 tens, which would be 20 tens. 20 tens is 200. So, 4×50 is 200.”</p> <p>Students might also compose/decompose numbers and use the Associative Property of Multiplication to find “friendly ways” to think through these problems:</p> <div style="text-align: right;"> $\begin{aligned} &30 \times 6 \\ &= 3 \times 10 \times 6 \\ &= 10 \times 18 && \text{(See 3.OA.5)} \\ &= 10 \text{ groups of } 18 && \text{(See 3.OA.1)} \\ &= 180 \end{aligned}$ </div> |
|--|--|

Number and Operations – Fractions⁵

⁵ Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Cluster

Develop understanding of fractions as numbers.

Vocabulary: equal parts, fraction, equal distance (intervals), equivalent fractions, reasonable, numerator, denominator, comparison, compare, $<$, $>$, $=$, justify, inequality

3.NF.1

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

The Standards have laid a foundation for fractions in Grades 1 and 2 by asking students to partition circles and rectangles into equal shares and use fraction *words* or *names* (not symbols, such as $\frac{1}{2}$) to describe those shares:

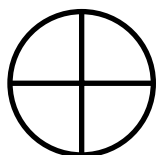
- * In 1st Grade, students partition circles and rectangles into two and four equal parts and describe the shares as halves and fourths (or quarters). **(1.G.3)**
- * In 2nd Grade, students partition circles and rectangles into two, three, or four equal parts and describe the shares as halves, thirds, and fourths. **(2.G.3)**

Research shows that “repeated halving” (cutting a whole in half, then cutting those halves in half, etc.) is a powerful strategy for helping students learn how to split one whole into smaller equal parts. The Standards incorporate this research to scaffold students’ work with fractions across Grades 1- 4:

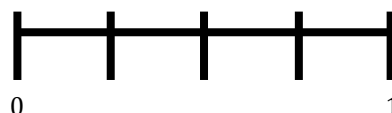
| Grade | Expectations |
|-----------------------|--|
| 1 st Grade | Halves and fourths (words/names, not symbols/fractions notation) |
| 2 nd Grade | Halves, thirds, and fourths (words/names, not symbols/fraction notation) |
| 3 rd Grade | Denominators of 2, 4, 8, 3 and 6 (words <i>and</i> symbols/fraction notation) |
| 4 th Grade | Denominators of 2, 4, 8, 16, 3, 6, 12, 5, 10, and 100 (words <i>and</i> symbols/fraction notation) |

In Grade 3, the emphasis is on splitting one whole (circle, rectangle, or measurement unit **(3.NF.2)**) into equal parts or “fair shares.” These models are often referred to as area models or measurement models. Set models (sets of individual objects) of fractions are deferred until Grade 4. (*Progressions for the CCSSM (Draft): Number and Operations – Fractions*, 3-5, Sept 2013, p. 3)

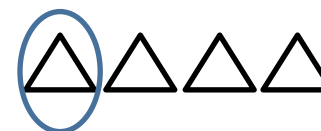
Area Model (3rd Grade)



Measurement Model (3rd Grade)



Set Model (4th Grade)



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3.NF.1 (cont'd)

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

Students' work with fractions in Grade 3 is grounded in pictures and models. In addition to area models and measurement models, Pattern Blocks, Cuisenaire Rods, Fraction Tiles, and Fraction Bars are all useful manipulatives that can help students learn about fractions with understanding.

Traditional fraction notation may be interpreted in terms of parts and wholes. The denominator tells us how many equal parts the whole is split into. The numerator tells us how many of those sized parts that we have (or are interested in, depending on the context of the problem).

Example: How could we describe the unshaded portion of this rectangle as a fraction?



Student: The rectangle is split into 4 equal parts, so the denominator is 4. Only one of the parts isn't shaded, so the numerator is 1. So we could say $\frac{1}{4}$ (one fourth) of the rectangle is unshaded.

Example: What fraction of that rectangle is shaded?

Student: The rectangle is split into 4 equal parts, or fourths. You just count up the shaded parts: one fourth, two fourths, three fourths. So, three fourths of the rectangle is shaded: $\frac{3}{4}$

It is essential for students to understand that fractional amounts *are in reference to a whole*. "One half" is "one half *of something*." "One fourth" is "one fourth *of something*." Teachers may need to set expectations for students to use clear and precise language (age appropriate but still mathematically accurate) when talking about fractions. For example,

Teacher: Two friends split a cookie so that they each got a fair share. How much did each friend get?

Student: A half.

Teacher: A half of what? A half of an apple? A half of a candy bar?

Student: No, a half *of the cookie*.

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3.NF.1 (cont'd)

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

In order for fractions to be compared meaningfully, they must reference the same whole. This concept is often not obvious to students in the beginning, but rich tasks and discussion prompts can help them realize the importance of this concept.

Example:

Erin ate half of a small pizza. Sam ate half of a large pizza. Did Erin and Sam eat the same amount of pizza?

Student: No! Half of a small pizza wouldn't be as much as half of an extra large pizza!

Teacher: But they're both halves?

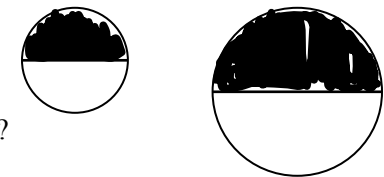
Student: But they're not halves of the same thing.

Teacher: Can you draw me a picture to show me what you're thinking?

Student: See? Half of a small pizza is less than half of a big pizza.

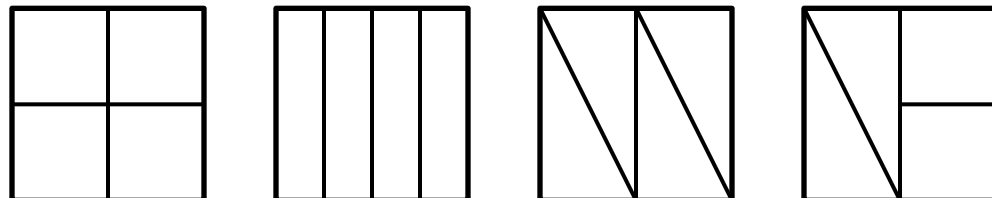
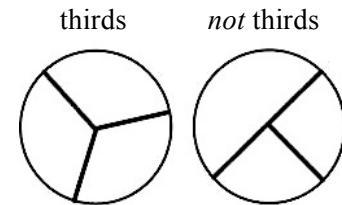
Teacher: So, we have to be really clear when describing fractions, don't we?

Student: Yeah. You gotta' say what the fraction *is a fraction of*. It matters.



Important ideas for conceptual understanding of fractions in Grade 3:

- Fractions refer to equal parts, equal shares, or "fair shares."
- The more equal parts the whole is split into, the smaller the parts are.
- The fewer equal parts the whole is split into, the bigger the parts are.
- Fractions reference *a whole*.
- We describe $\frac{3}{4}$ as "three fourths" of a whole, not as "three over four."
- Fractions don't have to be the same shape to represent the same amount. Here are several different ways to split the same square into four equal parts (fourths). Although they look different, each fourth represents the same fraction of the whole as any of the other fourths.



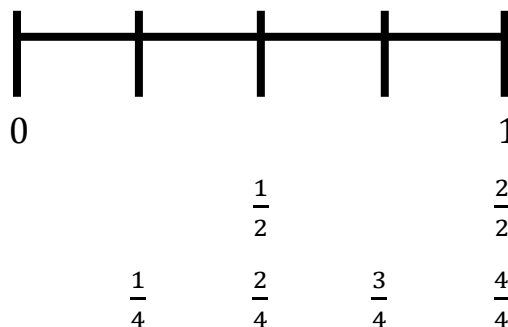
3.NF.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

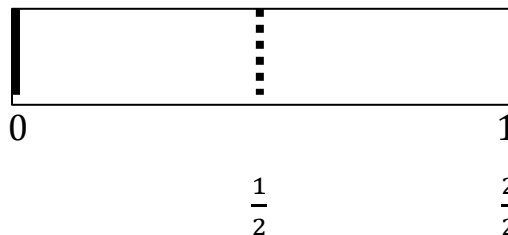
b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

This Standard builds on students' work with using a number line to represent whole numbers, as well as to model addition and subtraction of whole numbers (**2.MD.6**). In Grade 3, students now begin to examine parts of whole numbers and consider whole numbers as groups of parts (Ex: 1 whole can be described as 4 fourths. See **3.NF.3**.)



Exploring fractions with a number line diagram presents a new challenge for students in that the “whole” is not an *area* (such as a circle or rectangle). In a number line diagram, one whole is one unit of *distance* or *length*. Similarly, a fractional part represents part *of the distance* (or *length*), not an “area.”

“Hands on” experiences, such as folding strips of paper, can help students make sense of fractions in this context. Let’s take a strip of paper and label the ends 0 and 1 to show that the length of the strip is 1 whole unit of length.



If we fold the strip of paper end-to-end, we have now split the whole length into 2 equal parts. If we start at 0 and “slide” to the fold, we have traveled 1 of the 2 equal parts in the whole distance. So, we can label that distance as $\frac{1}{2}$. If we start at 0 and slide all the way down to the other end of the paper, we have traveled 2 of the 2 equal parts, or two halves: $\frac{2}{2}$ or 1 whole unit of length.

TEACHER NOTE: We must be clear that the “hash marks” which we traditionally label with the fractional amounts mark the end of a space or distance traveled. *The hash mark labeled “1/4” is not one fourth of the whole*; rather, it marks the distance of one of four equal spaces in the distance from 0 to 1. It is very important to emphasize that when we count fractional amounts on a number line model, we are counting the equal spaces (or the distance traveled) between 0 and 1; **we are not counting the “hash marks.”**

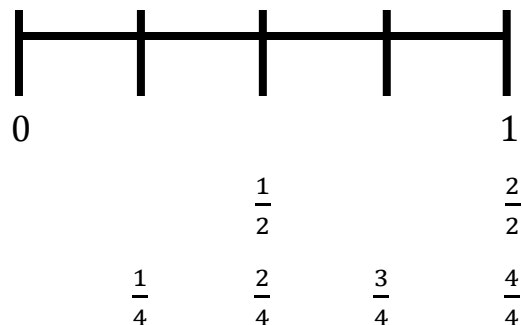
3.NF.3

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

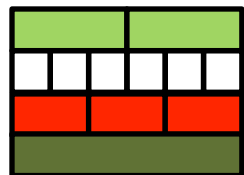
- Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **Recognize that comparisons are valid only when the two fractions refer to the same whole.**
- Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.
- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Students should understand that equivalent fractions represent the same amount in different ways. For example, in our number line model below, $\frac{1}{2}$ and $\frac{2}{4}$ both represent the same distance from 0. But they describe that distance in terms of different sizes and numbers of parts of the whole distance.



Students can also explore equivalent fractions using Cuisenaire Rods and Pattern Blocks. For example, if we let the dark green Cuisenaire Rod represent 1 whole, we can see the following relationships:



- 2 light green rods make a dark green rod. So, 1 light green rod represents $\frac{1}{2}$ of the whole.
- 6 white rods make a dark green rod. So, 1 white rod represents $\frac{1}{6}$ of the whole.
- 3 red rods make a dark green rod. So, 1 red rod represents $\frac{1}{3}$ of the whole.

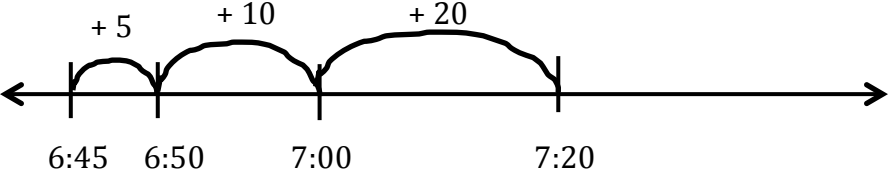
We can see that 2 white rods cover the same distance as 1 red rod. So, $\frac{2}{6}$ is equivalent to $\frac{1}{3}$, or $\frac{2}{6} = \frac{1}{3}$.

Building on that relationship, 4 white rods cover the same distance as 2 red rods. So, $\frac{4}{6}$ is equivalent to $\frac{2}{3}$, or $\frac{4}{6} = \frac{2}{3}$.

We can also see that 1 light green rod covers the same distance as 3 white rods. So, $\frac{1}{2} = \frac{3}{6}$.

This model also helps us see that 1 whole can be described as the sum of different parts: 2 halves, 6 sixths, or 3 thirds – or $1 = \frac{2}{2} = \frac{6}{6} = \frac{3}{3}$.

TEACHER NOTE: Please note the emphasis on visual fraction models within this Standard – *not calculations or conversions*. Over time, it is appropriate to ask students to look for patterns/relationships between equivalent fractions; but the emphasis in Grade 3 is to help them make sense of fraction relationships with visual references.

| Measurement and Data | |
|--|---|
| Cluster | |
| Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects. | |
| Vocabulary: estimate, time, minute, hour, elapsed time, measure, liquid volume, mass, standard units, metric units, gram (g), kilogram (kg), liter (l) | |
| <p>3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p> | <p>Using a number line model for elapsed time builds on students' work with using the number line to model and solve addition and subtraction problems in Grade 2 (2.MD.6).</p> <p><u>Example:</u> Daniel is in the 3rd Grade. He is supposed to be at his bus stop just outside of his house at 7:30 in the morning. It takes Daniel 5 minutes to wash his face, 10 minutes to get dressed, and 20 minutes to eat breakfast. If Daniel wakes up at 6:45 in the morning, will he be able to get to the bus stop in time?</p>  <p><u>Student:</u> I started at 6:45 because that's when he woke up. I jumped 5 minutes for washing his face and landed at 6:50. Then I jumped 10 minutes to show him getting dressed, and that put me at 7:00. Then I jumped 20 minutes to show him eating breakfast and landed on 7:20. So, yes, he's got 10 minutes to get to the bus stop.</p> |
| <p>3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).⁶ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.⁷</p> <p>⁶ Excludes compound units such as cm^3 and finding the geometric volume of a container.</p> <p>⁷ Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Table 2).</p> | <p>This Standard asks students to reason about the units of mass and volume using grams (g), kilograms (kg), and liters (l). Opportunities to weigh classroom objects and fill containers help students develop meaningful real-world references of "about how much" 1 gram, 1 kilogram, and 1 liter represent. This helps to develop "measurement sense" and also helps them self-correct if they make mistakes when solving word problems.</p> <p>Word problems should not expect students to convert between units.</p> <p><u>Example:</u></p> <p><u>Teacher:</u> I put one large paperclip on everyone's desks. Let's each pick up our paperclips and hold them in the palms of our hands. Would you describe the paperclip as really heavy, a little bit heavy, or not very heavy?</p> <p><u>Student:</u> Not very heavy.</p> <p><u>Teacher:</u> "Mass" is typically something we might talk about in science, but it's related to mathematics, too, because it's something that we can measure. Mass is a way of describing how much matter it takes to make something. This paperclip has a mass of about 1 gram. I'm going to bring around a box of 100 paperclips to each group. If one paperclip has a mass of about 1 gram, how much mass would the box have?</p> <p>(continued on next page)</p> |

| <p>3.MD.2 (<i>cont'd</i>) Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).⁶ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.⁷</p> | <p><u>Student:</u> About 100 grams.</p> <p><u>Teacher:</u> Take turns in your group picking up the box and holding it in your hands. I'm going to put 10 minutes on the timer. Each group is responsible for looking around the room and collecting 2 objects that weigh about 1 gram and 2 objects that weigh about 100 grams. When the timer goes off, I'm going to ask each group to share with the rest of the class what objects they found.</p> <p>The Standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth's surface, the distinction is not important (on the moon, an object would have the same mass, would weigh less due to the lower gravity). (<i>Progressions for the CCSSM (Draft): Geometric Measurement</i>, CCSS Writing Team, June 2012, p. 2)</p> | | | | | | | | |
|---|---|----------------------|--|-------|-----------|------|-------------------|-------------|--|
| Cluster | | | | | | | | | |
| Represent and interpret data. | | | | | | | | | |
| Vocabulary: scale, picture graph, scaled picture graph, bar graph, scaled bar graph, line plot, data | | | | | | | | | |
| <p>3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.</p> <p><i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i></p> | <p>This Standard builds on students' work with single-unit bar graphs and picture graphs in Grade 2 (2.MD.10). Scaled bar graphs and picture graphs require a new level of abstract thinking for students in that 1 object no longer represents thing; 1 object represents <i>several things</i>. However, the Standards have laid a foundation for asking students to think in terms of "groups" rather than individual items in Grade 2: skip-counting (2.NBT.2) and working with money (2.MD.8).</p> <p>One strategy for introducing scaled graphs to students is to choose data that is difficult to represent with single-unit representations and then use questioning techniques to help students figure out a more "efficient" way to represent the data. (Ideally, one of them will suggest "counting by fives" or a similar strategy.)</p> <p>A common mistake for students is to ignore the scale (or key) and to interpret the graph incorrectly. For example, in the picture graph below, some students would say that Juan read 3 <i>more</i> books than Nancy did, rather than the correct answer of 15 books. It is important that students have multiple opportunities over time to learn how to interpret scales and answer questions knowledgeably.</p> <table border="1" data-bbox="1121 1187 1644 1404"> <thead> <tr> <th colspan="2">Number of Books Read</th></tr> </thead> <tbody> <tr> <td>Nancy</td><td>★ ★ ★ ★ ★</td></tr> <tr> <td>Juan</td><td>★ ★ ★ ★ ★ ★ ★ ★ ★</td></tr> <tr> <td colspan="2">★ = 5 Books</td></tr> </tbody> </table> | Number of Books Read | | Nancy | ★ ★ ★ ★ ★ | Juan | ★ ★ ★ ★ ★ ★ ★ ★ ★ | ★ = 5 Books | |
| Number of Books Read | | | | | | | | | |
| Nancy | ★ ★ ★ ★ ★ | | | | | | | | |
| Juan | ★ ★ ★ ★ ★ ★ ★ ★ ★ | | | | | | | | |
| ★ = 5 Books | | | | | | | | | |

3.MD.4

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units — whole numbers, halves, or quarters.

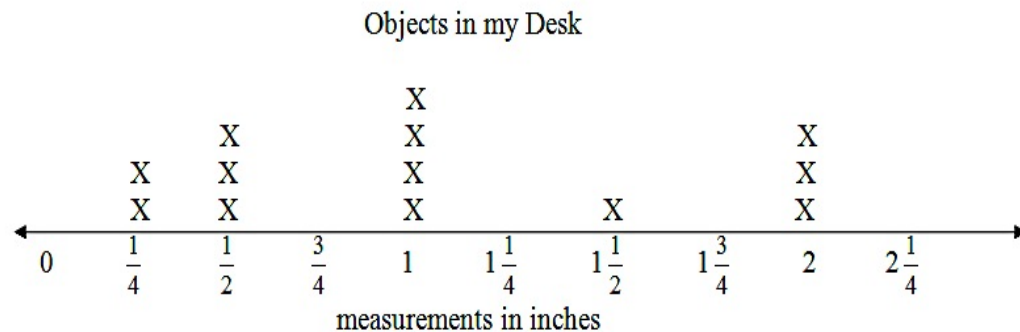
In Grade 2, students measured length in whole units and built line plots to represent that data. **(2.MD.9)** They extend that work here to include halves and fourths of length units, which corresponds to their work with representing fractional amounts on the number line **(3.NF.2)**.

Third graders need many hands-on opportunities to actively measure the length of various objects with physical rulers. **The focus of this Standard is on measuring and building line plots, not just responding to questions from pre-made line plots.** It is important to use meaningful formative assessments to see which students understand that measurement should begin at 0 on the ruler, not just “at the end of the ruler.”

It is important for students to understand that measuring “to the nearest half or fourth” of a unit can lead to a measurement in a whole unit. After all, 1 whole unit is 2 halves or 4 fourths. Students who lack a strong understanding of that relationship will often look for the closest number in which they can “see” the fourth or half in the number, even if the measurement is actually closer to a whole number (Ex: choosing $1\frac{1}{4}$ vs 1 because they can’t “see” the 4 fourths in 1 whole). These students need more experiences with models corresponding to **3.NF.3c**.

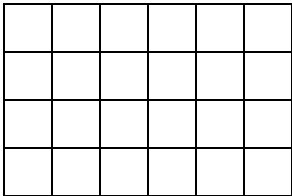
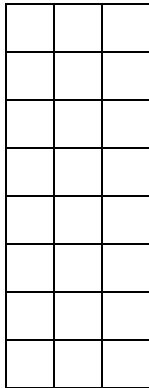
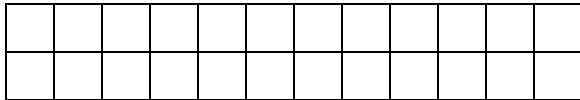
Example:

I have put a small basket of objects in the desk of one of the members of each group. I’m going to put 12 minutes on the timer. Your task is to work as a group to use your rulers and measure the length of each object to the nearest half-inch or quarter-inch. You will then make a line plot to record your measurements and write down 3 things that you notice about the data, based on your line plot. When the timer goes off, each group will present their results.



Student Responses:

- (1) The biggest objects that we had measured 2 inches. We didn’t have anything bigger than that.
- (2) The length that we had the most of was 1 inch. We had 4 things that were 1 inch long.
- (3) We measured 13 total objects.
- (4) We had 5 objects that measured less than 1 inch.
- (5) We had 4 objects that measured longer than 1 inch.
- (6) We had more objects that measured less than 1 inch than objects that measured more than 1 inch.

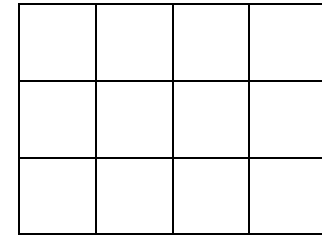
| Cluster | |
|---|---|
| Geometric measurement: understand concepts of area and relate area to multiplication and to addition. | |
| Vocabulary: area, side length, tiling, gap, overlap, square units, square cm, square m, square in, square ft, composing, decomposing | |
| <p>3.MD.5</p> <p>Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <p>b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</p> | <p>In Grade 2, students partitioned rectangles into rows and columns of same-sized squares and <u>counted</u> to find the total number of squares (2.G.2). They also used <u>addition</u> to find the total to find the total number of objects arranged in rectangular arrays within up to 5 rows and 5 columns (2.OA.4). Grade 3 builds on that prior knowledge and extends it to the 3rd Grade concepts of multiplication and area.</p> <p>Working with physical manipulatives (ex: square tiles) before transitioning to pictorial representations (ex: grid/graph paper) helps students move through the concrete → pictorial → symbolic stages of understanding.</p> <p>It is important for students to explore figures that may look different but have the same area.</p> <p><u>Example</u>: Look at the rectangles below. Before we measure anything, talk with your group and predict: Which of the following rectangles do you think will have the biggest area? Why do you think so?</p> <div style="display: flex; justify-content: space-around; align-items: center;">    </div> <p><u>Example Cont’d</u>: Count the number of unit squares it would take to cover each rectangles. Did your results fit your prediction? Why/why not?</p> <p>TEACHER NOTE: Student predictions to the first part of the task vary. Some students think that the rectangle on the left will be the biggest because it is “the fattest” or “the thickest.” Some students are influenced by the orientation of the middle rectangle and think that it is the “tallest” and thus will have more area. Other students will predict that the rectangle on the right will have the most area because it is “longer” than the others. It can be a very fun and engaging discussion when the students must confront the reality that all three rectangles have the same area (24 square units).</p> |

| | |
|---|---|
| <p>3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</p> | <p><u>Key concepts within this Standard:</u></p> <ul style="list-style-type: none"> Measuring area with different sized square units will affect the final measurement. <ul style="list-style-type: none"> The smaller the measurement unit, the more square units it will take to cover the area. The larger the measurement unit, the fewer square units it will take to cover the area. Ex: It will take more square centimeters than square inches to cover the same area because square centimeters are smaller than square inches. However, the actual area of the figure being measured <u>does not change</u>. The measurement of that area depends in on the units that were used. This is why it is so important to use units when exploring measurement – i.e., the answer is “24 <i>square inches</i>,” not just “24.” <p><u>TEACHER NOTE:</u> The area of a Base 10 Flat is 100 square centimeters or 1 square decimeter. This can provide another measurement unit to use in exploring area measurements/estimates.</p> <p><u>Example:</u> About how many Base 10 flats do you think it take to cover your desk? Now see how many flats it actually takes to cover your desk. Was your estimate close? If so, what advice would you give to someone who is trying to estimate area? If not, what might you do differently next time to make a better estimate?</p> <p>Students should also have opportunities to apply the strategy of measuring area in square units on non-rectangular and irregular figures. The following articles provide examples of classroom tasks that do so:</p> <p>Casa, Tutia M., Anne M. Spinelli, & M. Katherine Gavin. (2006). This about covers it! Strategies for finding area. <i>Teaching Children Mathematics</i>, 168-73.</p> <p>Moyer, P. (2001, September). Using representations to explore perimeter and area. <i>Teaching Children Mathematics</i>, 52-59.</p> |
| <p>3.MD.7 Relate area to the operations of multiplication and addition.</p> <p>a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</p> | <p><u>TEACHER NOTE:</u> The language we use in discussing area at this stage is very important. It is common for teachers (and students) to say, “Area equals length times width” or “Area is base times height.” <u>But this is not always true</u>. The areas of triangles, circles, trapezoids, (and other shapes) is <u>not</u> found by the formula $A = l \times w$. But once students “learn” that phrase, it is <u>very</u> difficult to get them away from it. It would be more accurate to say, “The area <i>of a rectangle</i> can be found by multiplying length times width.”</p> <p>This Standard works well with other 3rd Grade Standards addressing visual representations of multiplication, such as 3.OA.1 and 3.OA.3. Classroom discussions should help students build connections between multiple strategies, such as skip-counting (2.NBT.2) and recognizing multiplication as repeatedly adding equal groups (3.OA.1).</p> |

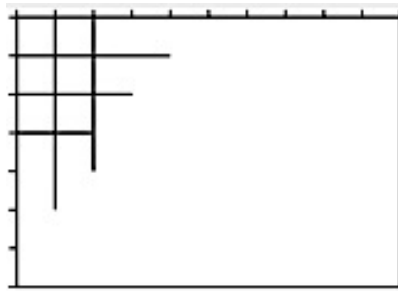
Example:

What is the area of this rectangle?

Student: I can cover the area in 12 square tiles, so the area is 12 square units. You could also look at it as 3 rows with 4 tiles in each row. That's like 3 groups of 4 square tiles, which is also 12 square tiles.



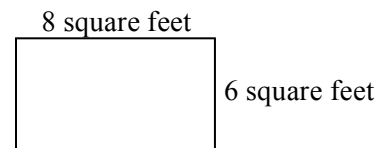
To help students move forward from depending on seeing each square unit individually, certain tasks can help students see the value/efficiency in using multiplication to calculate the area of a rectangular figure. To find the area of the incomplete array below, students *could* draw in all of the tiles and then count them individually. But that would take a long time. If we simply figure out the side lengths of the rectangle in units, we could multiply to figure out how many square units it must take to cover the entire figure.



(Progressions for the CCSSM (Draft): Geometric Measurement, CCSS Writing Team, June 2012, p. 17)

Example:

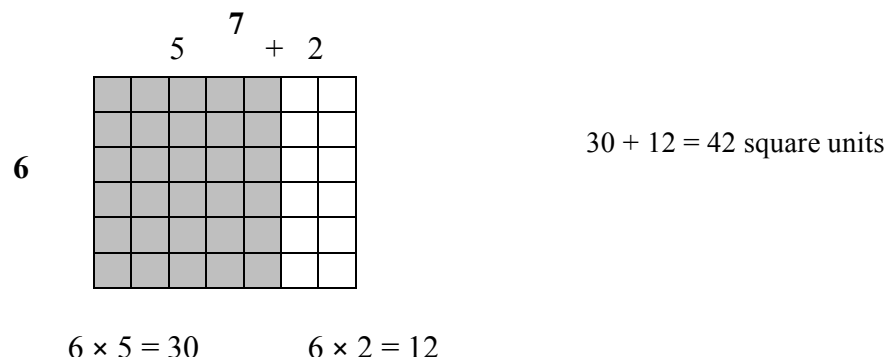
Drew wants to tile the bathroom floor using square tiles that measure 1 foot on each side. Below is a picture of Drew's bathroom floor. How many square foot tiles will he need to cover the floor?



b. Multiply side lengths to find areas of rectangles with whole-number side lengths (**where factors can be between 1 and 10, inclusively**) in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

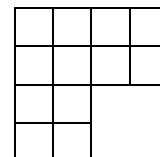
This Standard corresponds to **3.OA.5**. For example, in the picture below the area of a 6×7 figure can be determined by finding the area of a 6×5 and 6×2 and adding the two sums.



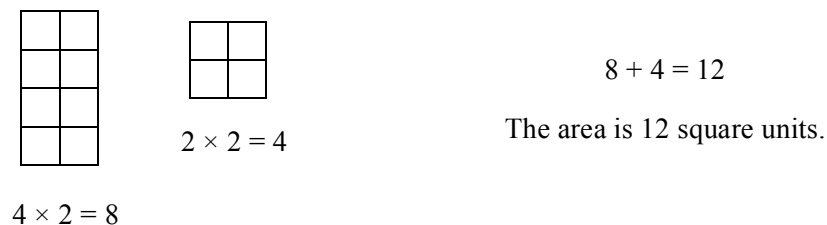
d. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. Recognize area as additive.

This standard uses the term “rectilinear.” A rectilinear figure is a polygon that has all right angles.

Example: What is the area of this figure?



Answer: We could count the number of square units in the figure. But we could also decompose (break apart) the figure into smaller pieces and use our work with arrays to find the area of the figure in a different way:



| Cluster | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|--|---|--------|-----------|-------|-----------|------------|-------|--------|--------|------------|-------|-------|--------|------------|-------|-------|--------|------------|-------|-------|--------|------------|-------|-------|--------|------------|--------|-------|--------|
| Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Vocabulary: perimeter, length units, area units, polygon, sides | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting (including, but not limited to: modeling, drawing, designing, and creating) rectangles with the same perimeter and different areas or with the same area and different perimeters. | <p>In this Standard, the emphasis is on helping students develop a conceptual understanding of perimeter and area, <i>not on learning formulas</i>. Students are not expected to work with formulas for perimeter and area until Grade 4 (4.MD.3).</p> <p>Students often confuse area and perimeter unless they are given meaningful experiences/references for these measurements. The article below provides an excellent example of a classroom task that incorporates the idea of “seating people at dinner tables” to help students deepen their understanding of perimeter and area:</p> <p>Moyer, P. (2001, September). Using representations to explore perimeter and area. <i>Teaching Children Mathematics</i>, 52-59.</p> <p><u>Example:</u> How many different rectangles can you find that have an area of 12 square units? (You may only use whole numbers in your dimensions.) Record the area, length, width, and perimeter of the rectangles you create in the table. What patterns do you notice?</p> <table><tr><th>Area</th><th>Length</th><th>Width</th><th>Perimeter</th></tr><tr><td>12 sq. in.</td><td>1 in.</td><td>12 in.</td><td>26 in.</td></tr><tr><td>12 sq. in.</td><td>2 in.</td><td>6 in.</td><td>16 in.</td></tr><tr><td>12 sq. in.</td><td>3 in.</td><td>4 in.</td><td>14 in.</td></tr><tr><td>12 sq. in.</td><td>4 in.</td><td>3 in.</td><td>14 in.</td></tr><tr><td>12 sq. in.</td><td>6 in.</td><td>2 in.</td><td>16 in.</td></tr><tr><td>12 sq. in.</td><td>12 in.</td><td>1 in.</td><td>26 in.</td></tr></table> <p>The patterns in the chart allow students to identify the factors of 12, connect the results to The Commutative Property of Multiplication, and discuss the differences in perimeter within the same area. It is a powerful concept for the students to realize that just because two rectangles have the same area, that does not mean that they have the same perimeter. (See 3.MD.5)</p> <p>It can be exciting to extend the problem above to areas that can be modeled as a square (ex: 36 square units). This can provide excellent discussions about squares as special types of rectangles, which connects to 3.G.1.</p> | Area | Length | Width | Perimeter | 12 sq. in. | 1 in. | 12 in. | 26 in. | 12 sq. in. | 2 in. | 6 in. | 16 in. | 12 sq. in. | 3 in. | 4 in. | 14 in. | 12 sq. in. | 4 in. | 3 in. | 14 in. | 12 sq. in. | 6 in. | 2 in. | 16 in. | 12 sq. in. | 12 in. | 1 in. | 26 in. |
| Area | Length | Width | Perimeter | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 sq. in. | 1 in. | 12 in. | 26 in. | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 sq. in. | 2 in. | 6 in. | 16 in. | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 sq. in. | 3 in. | 4 in. | 14 in. | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 sq. in. | 4 in. | 3 in. | 14 in. | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 sq. in. | 6 in. | 2 in. | 16 in. | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 sq. in. | 12 in. | 1 in. | 26 in. | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Geometry | |
|--|---|
| Cluster | |
| Reason with shapes and their attributes. | |
| Vocabulary: properties, attributes, features, quadrilateral, open figure, closed figure, vertex/vertices, triangle, circle, quadrilateral, rectangle, square, trapezoid, rhombus, rhombuses/rhombi, kite, equal area, whole, unit fraction | |
| <p>3.G.1</p> <p>Understand that shapes in different categories (e.g., rhombuses, rectangles, circles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p> | <p>In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate the family of shapes known as “quadrilaterals.”</p> <p>A quadrilateral is a closed figure with four straight sides. Students should have opportunities to explore and discuss why parallelograms, rectangles, rhombi, squares, trapezoids, and kites* have the same attributes as quadrilaterals and are thus considered “to be types of quadrilaterals” or “to belong to the quadrilateral family.” They should also explore other shapes that do not belong to the quadrilateral family and to discuss why these shapes do not have the same features as quadrilaterals (ex: hexagons and triangles).</p> <p><u>TEACHER NOTE:</u> It is important for students to have opportunities to explore and discuss how squares and rectangles are related. The mathematical attributes of a rectangle <i>do not include</i> “having two long sides and two short sides.” Those characteristics should not be taught as defining attributes of a rectangle. In its most general terms, a rectangle is a parallelogram that has 4 right angles. (By belonging to the “parallelogram family,” we know that a rectangle has two opposite pairs of parallel sides and two opposite pairs of congruent sides.) A square fits all of the characteristics of a rectangle. It is a <i>special type of rectangle</i> in that <u>all</u> of the sides of a square are congruent.</p> <div data-bbox="1087 883 1346 1023" data-label="Image"> </div> <p>It is also important to note that orientation of a figure does not change the figure itself. Given the shapes above, students often refer to the figure on the left as a square and the figure on the right as a diamond. <u>Both figures are squares</u>; the square on the right has just been rotated.</p> <p>Unfortunately, many “educational” materials refer to a rhombus, or even a rotated square, as a “diamond.” <u>“Diamond” is not a geometric term & should not be used to describe shapes.</u></p> <p>* Students do not necessarily have to be able to identify kites in Grade 3. However, it may be helpful to know that a kite <i>is</i> a geometric shape. A kite is a quadrilateral with (at least) two pairs of adjacent congruent sides and whose diagonals form right angles.</p> <div data-bbox="1535 1331 1923 1456" data-label="Image"> </div> |

(continued on next page)

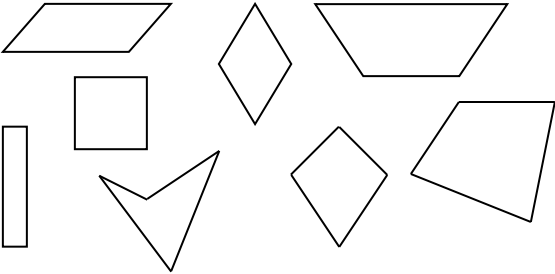
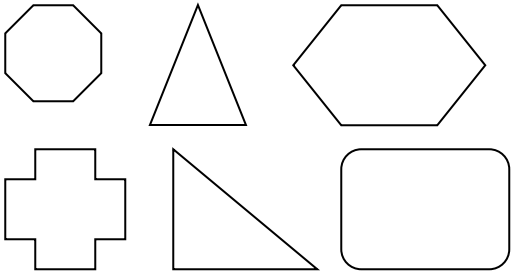
3.G.1 (cont'd)

Understand that shapes in different categories (e.g., rhombuses, rectangles, **circles**, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Research has shown that it is more helpful for students to see examples and non-examples of shapes, rather than to memorize definitions, when learning about shape properties and shape families.

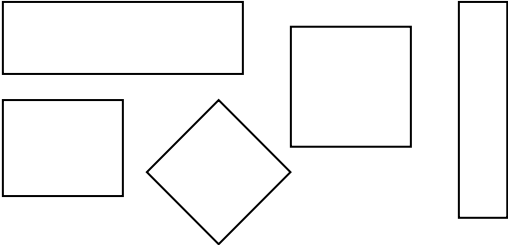
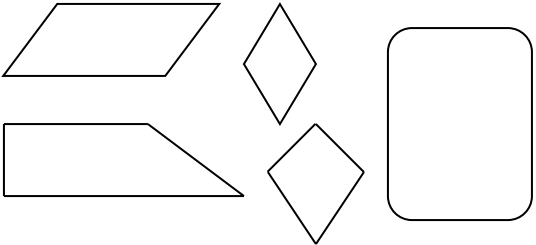
Example:

Look at how the shapes have been sorted into the two groups. Based on what you see, what do you think a shape has to have to be a quadrilateral?

| These are quadrilaterals: | These are not quadrilaterals: |
|--|---|
|  |  |

Similar “sort” explorations can be used to explore the relationships between rectangles and squares:

Look at how the shapes have been sorted into the two groups. Based on what you see, what do you think a shape has to have to be a rectangle? Or, how can you tell if a shape is not a rectangle?

| These are rectangles: | These are not rectangles: |
|--|---|
|  |  |

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(Examples on this page used with permission from The Center for Mathematics and Science Education at The University of Mississippi.)

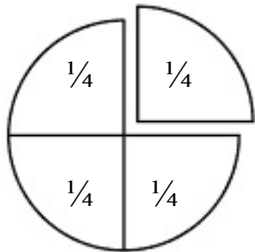
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| <p>3.G.1 (<i>cont'd</i>)</p> | <p><i>TEACHER NOTE:</i> In the U.S., the term “trapezoid” may have two different meanings. Research identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with <u>at least one pair of parallel sides</u>. With this definition, parallelograms, rectangles, squares, and rhombi fit that definition and can thus be considered as <i>types of trapezoids</i>. The exclusive definition states: A trapezoid is a quadrilateral with <u>exactly one pair of parallel sides</u>. With this definition, parallelograms and their subgroups do not fit the definition and thus are not considered to be types of trapezoids. (<i>Progressions for the CCSSM: Geometry</i>, The Common Core Standards Writing Team, June 2012.)</p> | | | | | | | | | | | | |
| <p>3.G.2</p> <p>Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.</p> <p><i>For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</i></p> | <p>This Standard clearly connects to 3.NF.1. In Grade 3, students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths. The whole can be a shape such as a circle or rectangle. Pattern blocks are also very useful for modeling these concepts.</p> <p><u>Examples:</u></p> <p>This figure was partitioned/divided into four equal parts. Each part is $\frac{1}{4}$ of the total area of the figure.</p> <div></div> <p>Given a shape, students partition it into equal parts in several different ways, recognizing that these parts all have the same area:</p> <div><div><table><tr><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td></tr><tr><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td></tr></table></div><div><table><tr><td>$\frac{1}{4}$</td></tr><tr><td>$\frac{1}{4}$</td></tr><tr><td>$\frac{1}{4}$</td></tr><tr><td>$\frac{1}{4}$</td></tr></table></div></div> <div><table><tr><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td></tr></table></div> | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
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Table 2: Common Multiplication and Division Situations¹

| | Unknown Product $3 \times 6 = ?$ | Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$ | Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$ |
|---|--|--|---|
| Equal Groups | There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| Arrays ² , Area ³ | There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| Compare | A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| General | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |

¹The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

²The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

³Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

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