Fluency Expectations or Examples of Culminating Standards
- **6.NS.2:** *Fluently* divide multi-digit numbers using the standard algorithm.
- **6.NS.3:** *Fluently* add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

The following Standards have changes from the 2015-16 MS College- and Career-Readiness Standards:

Significant Changes (ex: change in expectations, new Standards, or removed Standards)
6.NS.9
6.SP.5c

Slight Changes (slight change or clarification in wording)
none

Throughout the 2016 Mississippi College- and Career-Readiness Standards for Mathematics Grades [6-8] Standards, the words fluency and fluently will appear in bold, italicized, and underlined font (for example: ***fluently***). With respect to student performance and effective in-class instruction, the expectations for mathematical fluency are explained below:

Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Wherever the word ***fluently*** appears in a MS CCR content standard, the word means quickly and accurately. It is important to understand that this is not explicitly tied to assessment purposes, but means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn't halting, stumbling, or reversing oneself.

A key aspect of fluency is this sense that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

2016 Mississippi College- and Career-Readiness Standards for Mathematics, p. 19

Ratios and Proportional Relationships	
Cluster	
Understand ratio concepts and use ratio reasoning to solve problems.	
Vocabulary: ratio, equivalent ratios, rate, unit rate, tape diagram, double number line, part-to-part, part-to-whole, percent.	

Standard	Clarifications
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For Example,* *"The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak."* *"For every vote candidate A received, candidate C received nearly three votes."*	A ***ratio*** is a multiplicative comparison of two quantities or measures. The comparison can be part-to-whole (ratio of orange concentrate to orange juice) or part-to-part (ratio of orange concentrate to water). As students begin to explore ratios, they build off of their understanding of multiplication developed in elementary school. Students start to exercise and sharpen their ability to reason multiplicatively, *The orange concentrate is how many times greater than the amount of water? The amount of orange concentrate is what part or what fraction of the orange juice?* Students also begin to apply their multiplicative reasoning by thinking in terms of "*groups of*" as they begin to iterate and partition composed units they have built to create equivalent ratios. Example 1: Lets look at the ingredients for a recipe to make orange juice. Orange Orange Orange Orange Orange Teacher: Write the ratio of orange concentrate to water. Student: The ratio of orange concentrate to water is 2 parts orange to 3 parts water. *This comparison could be expressed in any of the following forms:* $\frac{2}{3}$ *, 2 to 3, or 2:3.* It is important that teachers help students to understand that ratios and fractions are not the same thing. While ratios can be recorded in fraction form, not all ratios are a part-to-whole comparison. A teacher shouldn't assume that a student has developed an understanding of ratios just because they are able to present them in the form of $\frac{2}{3}$. Remember forming a ratio is a *cognitive task-not a writing task.* When first working with ratios, it may benefit students to record them as 2:3 or 2 to 3 until they can differentiate between a fraction and a ratio. The next example can exercise students' ability to reason with ratios.

Example 2:

Teacher: Does a batch of orange juice made with 2 cans of orange concentrate and 3 cans of water taste equally orangey, more orangey, or less orangey than a batch made with 6 cans of orange concentrate and 9 cans of water?

Student 1: The second batch would taste more orangey.

Teacher: Tell me a little more about your thinking.

Student 1: Well, both numbers are bigger, and it would taste more orangey because you have more orange concentrate.

Teacher: Does anyone else have an idea they would like to share?

Student 2: I think that they would both taste the same because 6 cans of orange concentrate and 9 cans of water are three groups of the original recipe 2:3. So when you make the second batch of orange juice, you are just repeating the recipe three times. You get three times the amount of the first batch, but the taste would remain the same because the recipe remains the same.

Student 1: I don't get it. Could you show me what you are talking about?

Student 2: Sure I drew a picture to help me think.
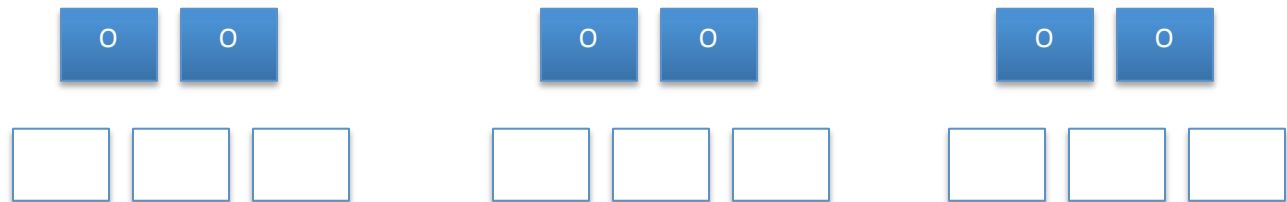
Student 1: Now I see! There are three groups of 2 which totals to 6 and then there are also 3 groups of 3 and that totals to 9. Even though there is more orange concentrate and more water, the recipe was just repeated so both batches would taste the same!

Adapted from *Developing Essential Understanding of Ratios, Proportions & Proportional Reasoning, NCTM*

	While working with tables of quantities in equivalent ratios (ratio tables), students should practice using and understanding important ratio and rate language. It is important for students to FOCUS on the meaning of the terms "for every," "for each," "for each 1," and "per" because these equivalent ways of stating ratios and rates are at the heart of understanding the STRUCTURE in these tables providing a foundation for learning about proportional relationships in **Grade 7.** Students should be able to identify and describe any ratio using "For every _____ , there are _____" In the example above, the ratio could be expressed saying, "For every 2 cans of orange concentrate, there are 3 cans of water." See article below for good instructional tasks that support this Standard. Tobias, Jennifer and Andreasen, Janet B. (2013) Developing Multiplicative Thinking from Additive Reasoning. *Teaching Children Mathematics,* 20(2), 102 -109.
6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For Example,* *"This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar."* *"We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* *Expectations for unit rates in this grade are limited to non-complex fractions.*	A ***rate*** is typically defined as a comparison of two quantities of different units (e.g. miles to hours or beats to minutes), in contrast to ***ratio***, which is often defined as a comparison of two quantities of like units, (cups to cups). A ***rate*** could also be defined as signifying a set of infinitely many equivalent ratios. *Thompson* (1994) A ***unit rate*** expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per unit of time (ex: hour, second). Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (i.e., miles / hour and hours / mile) are reciprocals as in the second example below. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule. **In 6th grade, students are NOT expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.** Example 1: Two football teams ordered pizza before the game. Team A ordered enough so that every 3 players will have 2 pizzas. Team B ordered enough so that there would be 3 pizzas for every 5 players. **Did Team A or Team B have more pizza per player?** Both teams claim they had more pizza than the other. What would you do to help the teams figure out who had more pizza? This can be modeled as shown below to show that there is $\frac{2}{3}$ of a pizza for each player on Team A and $\frac{3}{5}$ of a pizza for each player on Team B.
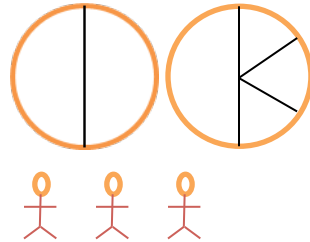
Team A   **Team B**
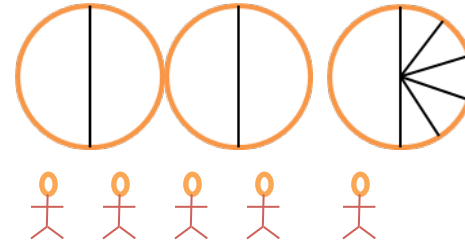
Teacher: I love that you have drawn a picture to show your thinking! Tell me how your picture helped you to solve this problem.

Student: Well, since Team A ordered two pizzas for three players, I drew the pizzas and I divided them in half. I saw that there were four halves. That gives each of the three players exactly $\frac{1}{2}$ of a pizza. Then I had to figure out how to split up the last half. I cut it into three slices. Since I cut $\frac{1}{2}$ into three pieces, each piece represents $\frac{1}{6}$. So each player on Team A can half $\frac{1}{2} + \frac{1}{6}$, or $\frac{2}{3}$ of a pizza.

Since Team B ordered three pizzas for every 5 players, I drew three pizzas and divided them in half. This gave me six halves. That means each of the 5 players can have half a pizza. Then there was still one half left. I cut it into five pieces. Since I cut half of a pizza into five pieces, each piece represents $\frac{1}{10}$. So each player on Team B can have $\frac{1}{2} + \frac{1}{10}$, or $\frac{3}{5}$ of a pizza.

Teacher: So which team served more pizza to each player?

Student: I can see from my picture that Team A served more pizza to each player. Both teams have enough to give half a pizza to each player, but in my picture I can see that $\frac{1}{6}$ is a larger amount than $\frac{1}{10}$.

Example 2:
A racecar travels 15 miles every 6 minutes. How far can the racecar travel in 15 minutes?

Solution: Jack can travel 5 miles in 1 hour written as $\frac{5mi}{1hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{1/5hr}{1mi}$. Students can represent the relationship between 20 miles and 4 hours.
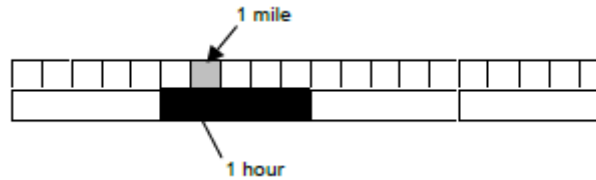
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	Ratios and rates can be used in ratio tables, graphs, tape diagrams, and double number lines to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using **multiplicative reasoning**. To aid in the development of proportional reasoning the cross-product algorithm is **not** expected at this level. When working with ratio tables and graphs, **whole number** measurements are the expectation for this standard. ***This is crucial foundational work to be built upon in 7th grade.***
a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	
	Progressions for the CCSSM (Draft): 6-7, Ratios and Proportional Relationships, December 26, 2011, p. 7

Example 1:

At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54?

Student: To find the price of 1 book, divide $18 by 3. One book costs $6. To find the price of 7 books, multiply $6 (the cost of one book) times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54.

Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (i.e. $1 \cdot 7 = 7$; $6 \cdot 7 = 42$). Red numbers indicate solutions.

| Number of Books (n)	Cost (C)
1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

Number of Books (n)	Cost (C)
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. **Writing equations is foundational for work in 7th grade.** For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$.

The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.

Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:

Books Unlimited
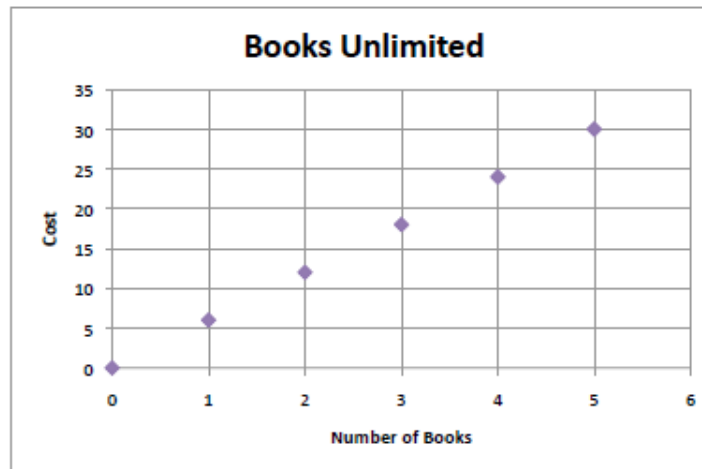
Example 2:

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.

Slimy Gloopy mixture is made by mixing glue and liquid laundry starch in a ratio of 3 to 2. How much glue and how much starch is needed to make 85 cups of Slimy Gloopy mixture?

Progressions for the CCSSM (Draft): 6-7, Ratios and Proportional Relationships, December 26, 2011, p. 7

Student:

Glue:

Starch:

85 cups

5 parts ➔ 85 cups

1 part ➔ 85 ÷ 5 = 17 cups

3 parts ➔ 3 • 17 = 51 cups

2 parts ➔ 2 • 17 = 34 cups

51 cups glue and 34 cups starch are needed.

Example 3:
Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Student 1: I added 9 feet and 15 feet to total 24 feet. So to find the missing value, I added 3 yards and 5 yards and that gave me a sum of 8 yards.
Student 2: I multiplied 3 feet by 8 to obtain a product of 24. Therefore 1 yard multiplied by 8 would give me 8 yards.
Student 3: I think I did something similar to Student 2, I saw that 6 feet multiplied by four would yield a product of 24. So then I knew to multiply 2 yards by 4. That gave me a product of 8 yards.

Example 4:
Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

There are several strategies that students could use to determine the solution to this problem
- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

b. Solve unit rate problems including those involving unit pricing and constant speed.

For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Example 1:

In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts?

Student 1: Since the ratio of peanuts to chocolate is 3:2, I drew a model to represent 3 parts to 2 parts. I know there are 9 cups of peanuts total.
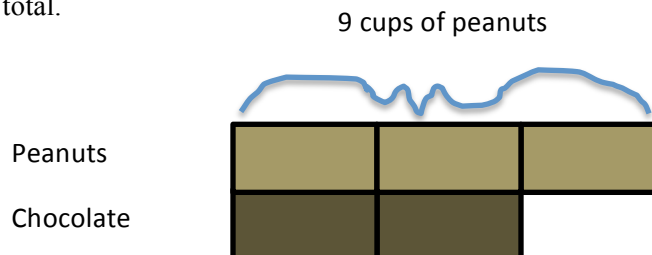
Since I know there are 9 cups of peanuts total, I divided them into 3 equal groups. This gave me 3 groups of 3. For every 3 cups of peanuts there are 2 cups of chocolate. So I drew another model below to represent the total amount of chocolate and peanuts needed for the trail mix. I can see that if there are 9 cups of peanuts, there will be 6 cups of chocolate.
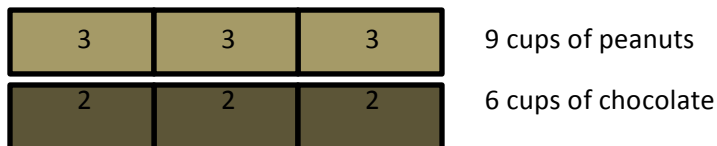
Student 2:

I created a table. I found the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, I multiplied the unit rate by nine ($9 \cdot \frac{2}{3}$), giving 6 cups of chocolate.

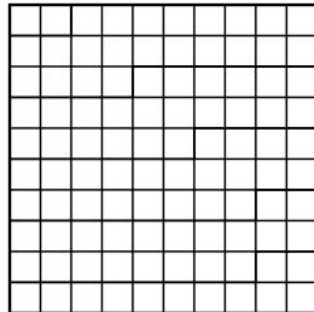| Peanuts	Chocolate
3	2

	Example 2: If steak costs \$2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer. Student: The unit rate is \$2.25 per pound so I multiplied \$2.25 x 0.8 to get \$1.80 per 0.8 lb of steak.
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	**Students are introduced to percentages for the first time in Grade 6.** Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percentages. Students use ratios to identify percentages. Teacher: Let's look at the flat in our Base 10 blocks. Let's let **one flat** represent **100 units.** Let's let **100 units** represent the **whole.** Teacher: Shade in one long. Teacher: How many units did you shade in? Student: I shaded 10 units. Teacher: The term "percent" is based on the idea of "per hundred". What percent of the units did you shade in? Student: Since I shaded 10 units out of 100 total units, I must have shaded 10 percent of the whole. Teacher: Now if I asked you to shade in **one half** of the whole, what would you shade in on the flat? Student: I would shade in 5 longs. Teacher: Explain how you know you shaded in one half of the whole flat. Student: 10 longs make a whole flat and 5 is half of 10. Teacher: How can we use your answer to describe "one half" as a percent? Student: There are 10 units in each of the five longs I shaded. 5 groups of 10 units would give me 50 units total. If a percent is the total per 100, that means $\frac{50}{100}$ would be 50%.

This would be a great time to fold in converting fractions to decimals as you introduce percentages.

Example 2:
What is 40% of 30?

There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.
See the weblink below for more information.
http://illuminations.nctm.org/LessonDetail.aspx?id=L249

Example 3:
What percent is 12 out of 25?
Building off of their work with ratios, one possible solution method is to set up a ratio table:

| Part	Whole
12	25
?	100

Student: I multiplied 25 by 4 to get 100. Multiplying 12 by 4 will give me 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.

Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

Students also determine the whole amount, given a part and the percent.

Example 4:
If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream?
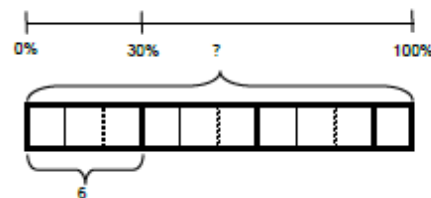
Student: I drew a picture to represent 100%. Each box represents   s
10 %. I know that 30% of the class is 6 students. This means that each of the boxes between 0% and 30% would have to be 2 students. Three groups of 30% would be 90%. Three groups of 6 students would be equivalent to 18 students.10% more would be 2 more students. 18 students plus 2 more students would be a total of 20 students in

Mrs. Rutherford's class.

Example 5:
A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals $450 for this month, how much interest would you have to be paid on the balance?

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get $76.50.

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter, and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, $\frac{12in}{1ft}$ is a conversion factor since the numerator and denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1ft}{12in}$ allowing for the conversion ratios to be expressed in a format so that units will "cancel."

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.

Example 1:
How many centimeters are in 7 feet, given that 1 inch ≈ 2.54 cm?

Solution:
7 feet x 12 inches x 2.54 cm = 213.36 cm

This is the conversion reference sheet provided to 6th graders on the current state math test. Students are expected to know conversion relationships inot on this table (ex: 12 in = 1 ft).

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 1000 cubic centimeters
		1 liter = 0.264 gallon

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System	
Cluster	
Apply and extend previous understands of multiplication and division to divide fractions by fractions.	
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: reciprocal, multiplicative inverses, visual fraction model.	

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.)* *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?* *How many 3/4-cup servings are in 2/3 of a cup of yogurt?* *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*	**Students need extensive experience in modeling fractional amounts using pictures and models before attempting traditional procedures manipulating fractional amounts. Number line, bar models or Cuisenaire Rods are very helpful for these representations.** In 5th grade students divide unit fractions by whole numbers and divide whole numbers by unit fractions, 5.NF.7. Students continue to develop this concept by using visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems. The Standards focus on two distinct models of division: "partitioning" (or "partitive") models and "measurement" (or "repeated subtraction") models. These problems are also described in Table 2. In a <u>partitioning model</u> of division ("Group Size Unknown"), the total number of objects is known and the total number of (desired) groups is known. The unknown (and usual focus of the question) is how much or how many objects should be in each group so that each group has the same amount. Example of Partitioning Story Problem: Zoe is having a tea party for her dolls. She has 4 dolls and 12 cookies. How many cookies should she give each doll so that they each get a fair share? The number sentence should reflect the actions taken to model and solve the problem. For "Zoe's Tea Party," this would be 12 ÷ 4 = 3. (12 cookies split into 4 equal groups puts 3 cookies in each group.) In a <u>repeated subtraction model</u> of division ("Number of Groups Unknown"), the total number of objects is known and the total number of objects in each (desired) group is known. The unknown (and usual focus of the question) is how many equal-sized groups will be made. Example of a Repeated Subtraction Story Problem: Josh is making favors for his birthday party. He has 14 mini candy bars, and he wants to put 2 bars in each bag. How many bags does Josh need? The number sentence should reflect the actions taken to model and solve the problem. For "Josh's Party," this would be 14 ÷ 2 = 7. (14 split into groups with 2 in each group makes 7 groups.)

One strategy for building off of students' experience in 5th grade would be to begin with unit fractions as divisors and then move to using non-unit fractions as divisors. Also to start, dividing fractions with like denominators before moving to fractions with unlike denominators can help reinforce the idea of the numerator (number of parts) vs. denominator (size of parts).

Example 1:

$\frac{6}{5} \div \frac{1}{5} =$ _____

Teacher: How many $\frac{1}{5}$ are in $\frac{6}{5}$?
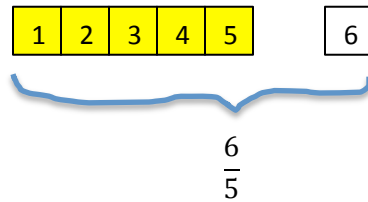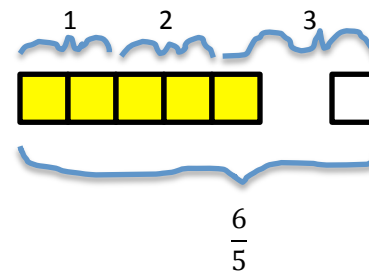

$$\frac{6}{5}$$

Student: I used the C-Rods and chose the yellow rod because it is 5 units in length. Then I needed to add one-fifth of the yellow rod to represent a total of $\frac{6}{5}$. I used the white unit rod to represent $\frac{1}{5}$ because five white rods are equal to the length of the yellow rod. I can see that there are 6 units of $\frac{1}{5}$ in $\frac{6}{5}$.   So, $\frac{6}{5} \div \frac{1}{5} = 6$

Example 2:

$\frac{6}{5} \div \frac{2}{5} =$ ____

Teacher: How many groups of $\frac{2}{5}$ are in $\frac{6}{5}$?
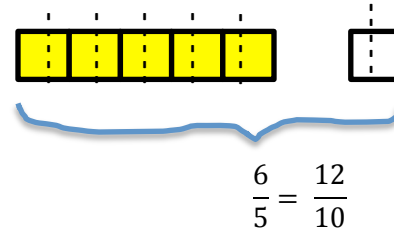

$$\frac{6}{5}$$

Student: I used the C-Rods and chose the yellow rod because it is 5 units in length. Then I needed to add one-fifth of the yellow rod to represent a total of $\frac{6}{5}$. I used the white unit rod to represent $\frac{1}{5}$ because five white rods are equal to the length of the yellow rod. I grouped together every two $\frac{1}{5}$ that I saw. I counted a total of 3 groups of $\frac{2}{5}$. I can see that $\frac{6}{5} \div \frac{2}{5} = 3$

Example 3:

$$\frac{6}{5} \div \frac{1}{10} = \underline{\hspace{2cm}}$$

<u>Teacher:</u> How many $\frac{1}{10}$ are in $\frac{6}{5}$?

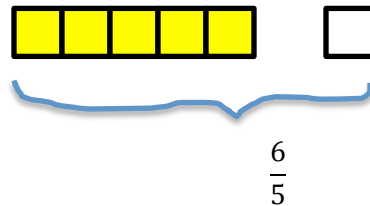

$$\frac{6}{5}$$

$$\frac{6}{5} = \frac{12}{10}$$

<u>Student:</u> I used the C-Rods and chose the yellow rod because it is 5 units in length. Then I needed to add one-fifth of the yellow rod to represent a total of $\frac{6}{5}$. I used the white unit rod to represent $\frac{1}{5}$ because five white rods are equal to the length of the yellow rod. I divided each unit rod of $\frac{1}{5}$ in half so that I would be able to see how many $\frac{1}{10}$ are in $\frac{6}{5}$. I counted each $\frac{1}{10}$ for a total of 12. I can see that $\frac{6}{5} \div \frac{1}{10} = 12$.

Scaffolding through the example problems above will lead students to learn to divide fractions using the unit fraction reasoning that is the foundation of all fraction understandings as laid out in our standards. (Used with permission from The Center for Mathematics and Science Education at The University of Mississippi.)

<u>Additional Examples of 6.NS.1</u>
Example 1:
A serving is $\frac{2}{5}$ of a brownie. There are 3 brownies. How many servings are there?

Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "How many groups of $\frac{2}{5}$ would it take to make 3?" One possible visual model would begin with three wholes and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students should be able to interpret the solution, explaining how division by fifths can result in an answer with halves.

This section represents half of two-fifths
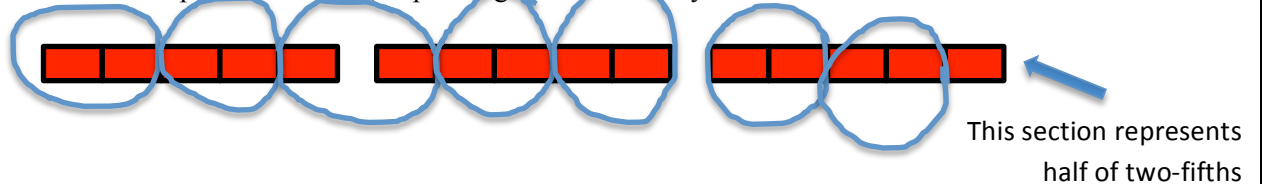
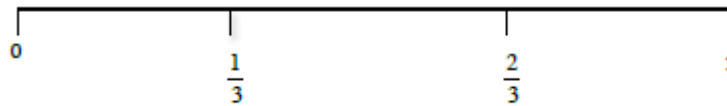Students should interpret contextual problems for fraction division problems. For example, consider the problem $\frac{2}{3} \div \frac{1}{6}$ set in context:

Example 2:

Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

This problem can be modeled using a number line.

    a.   Start with a number line divided into thirds.

    b.   The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.
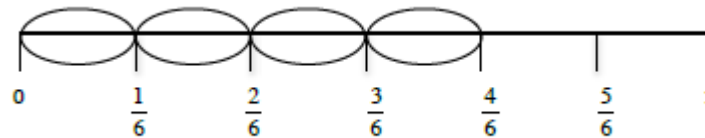
    c.   Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Example 3:

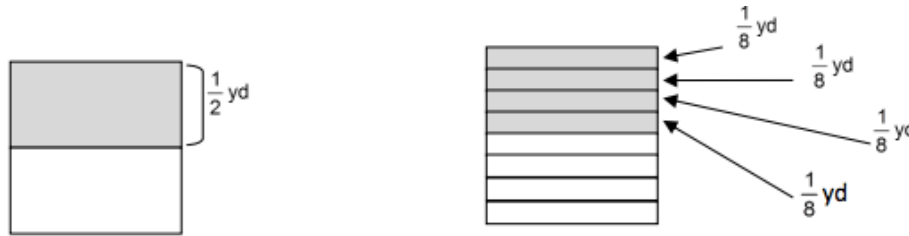Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.

Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?
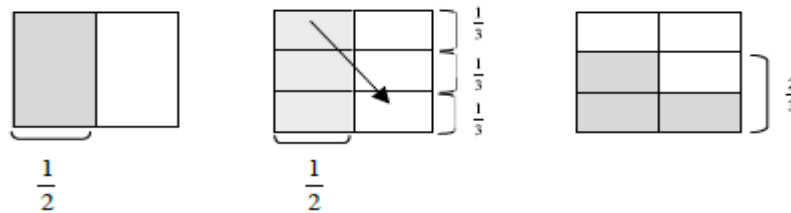
Explanation of Model:

The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cup horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only $\frac{3}{4}$ of the recipe can be made.

Cluster	
Compute fluently with multi-digit numbers and find common factors and multiples.	
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: multi-digit and algorithm.	

| **6.NS.2**

 Fluently divide multi-digit numbers using the standard algorithm. | In the elementary grades, students were introduced to division through **concrete models and various strategies to develop an understanding of this mathematical operation** (limited to 4-digit numbers divided by 2-digit numbers). Students will have had SOME exposure to the standard algorithms in 5th grade but are not expected to be fluent. In 6th grade, students become fluent in the use of the standard division algorithm, continuing to use their understanding of place value to describe what they are doing. Place value has been a major emphasis in the elementary standards. This standard is the end of this progression to address students' understanding of place value.

 Example 1:
 When dividing 32 into 8456, students should say, "there are 200 thirty-twos in 8456" as they write a 2 in the quotient. They could write 6400 beneath the 8456 rather than only writing 64. |

$$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$$
There are 200 thirty twos in 8456.

$$\begin{array}{r} 2 \\ 32\overline{)8456} \\ 6400 \\ \hline 2056 \end{array}$$
200 times 32 is 6400.
8456 minus 6400 is 2056.

$$\begin{array}{r} 26 \\ 32\overline{)8456} \\ 6400 \\ \hline 2056 \end{array}$$
There are 60 thirty twos in 2056.

$$\begin{array}{r} 26 \\ 32\overline{)8456} \\ 6400 \\ \hline 2056 \\ 1920 \\ \hline 136 \end{array}$$
60 times 32 is 1920.
2056 minus 1920 is 136.

$$\begin{array}{r} 264 \\ 32\overline{)8456} \\ 6400 \\ \hline 2056 \\ 1920 \\ \hline 136 \\ 128 \end{array}$$
There are 4 thirty twos in 136.
4 times 32 is 128.

$$\begin{array}{r} 264 \\ 32\overline{)8456} \\ 6400 \\ \hline 2056 \\ 1920 \\ \hline 136 \\ 128 \\ \hline 8 \end{array}$$
The remainder is 8. There is not a full thirty two in 8; there is only part of a thirty two in 8.

This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. There is ¼ of a thirty two in 8.

8456 = 264 * 32 + 8

| 6.NS.3	Procedural fluency is defined by the Common Core as "skill in carrying out procedures flexibly, accurately,
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	efficiently and appropriately." In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals were introduced in 5th grade (decimals to the hundredths place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, **students** become fluent in the use of the standard algorithms of each of these operations. They will have had SOME exposure to the standard algorithms in 5th grade but are not expected to be fluent until the end of 6th grade.
	The use of estimation strategies supports student understanding of decimal operations.
	Example 1:
	First estimate the sum of 12.3 and 9.75.
	Solution: An estimate of the sum would be 12 + 10 or 22. Students could also state if their estimate is high or low.
	Answers of 230.5 or 2.305 indicate that students are not considering place value when adding.

Cluster
Compute fluently with multi-digit numbers and find common factors and multiples.
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: greatest common factor, least common multiple, prime numbers, composite numbers, relatively prime, factors, multiples, distributive property, prime factorization.

6.NS.4	In elementary school, students identified primes, composites and factor pairs (4.OA.4). In 6th grade students
Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	will find the greatest common factor of two whole numbers less than or equal to 100.
	For example, the greatest common factor of 40 and 16 can be found by
	1) listing the factors of 40 (1, 2, 4, 5, 8, 10, 20, 40) and 16 (1, 2, 4, 8, 16), then taking the greatest common factor (8). Eight (8) is also the largest number such that the other factors are **relatively prime** (two numbers with no common factors other than one). For example, 8 would be multiplied by 5 to get 40; 8 would be multiplied by 2 to get 16. Since the 5 and 2 are relatively prime, then 8 is the greatest common factor. If students think 4 is the greatest, then show that 4 would be multiplied by 10 to get 40, while 16 would be 4 times 4. Since the 10 and 4 are not relatively prime (have 2 in common), the 4 cannot be the greatest common factor.
	(*Examples continued on next page.*)

2) listing the prime factors of 40 (2 • 2 • 2 • 5) and 16 (2 • 2 • 2 • 2) and then multiplying the common factors (2 • 2 • 2 = 8).

(2 • 2 • 2 = 8).

The product of the intersecting numbers is the GCF
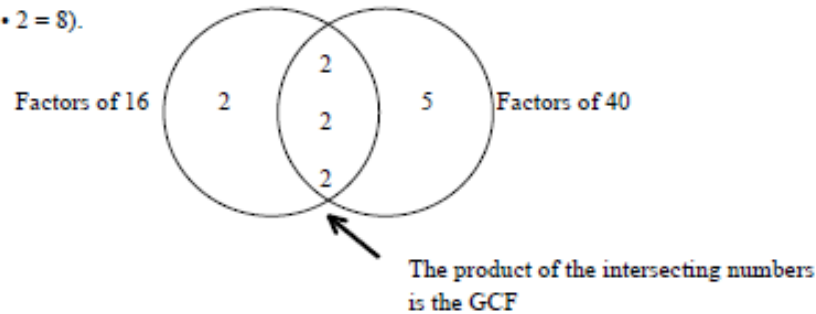
Students also understand that the greatest common factor of two prime numbers is 1.

Example 1:

What is the greatest common factor (GCF) of 18 and 24?

Solution: $2 \times 3^2 = 18$ and $2^3 \times 3 = 24$. Students should be able to explain that both 18 and 24 will have at least one factor of 2 and at least one factor of 3 in common, making 2×3 or 6 the GCF.

Given various pairs of addends using whole numbers from 1-100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.

Example 2:
Use the greatest common factor and the distributive property to find the sum of 36 and 8.
36 + 8 = 4 (9) + 4 (2)
44 = 4 (9 + 2)
44 = 4 (11)
44 = 44

Students should be able to find the <u>least common multiple</u> of two whole numbers less than or equal to twelve. For example, the least common multiple of 6 and 8 can be found by

1) listing the multiplies of 6 (6, 12, 18, 24, 30, …) and 8 (8, 26, 24, 32, 40…), then taking the least in common from the list (24); or

2) using the prime factorization.
Step 1: find the prime factors of 6 and 8.
$6 = 2 \cdot 3$
$8 = 2 \cdot 2 \cdot 2$

Step 2: Find the common factors between 6 and 8. In this example, the common factor is 2

Step 3: Multiply the common factors and any extra factors: $2 \cdot 2 \cdot 2 \cdot 3$ or 24. (One of the twos is in common; the other twos and the three are the extra factors.)

Example:
The elementary school lunch menu repeats every 20 days; the middle school lunch menu repeats every 15 days. Both schools are serving pizza today. In how may days will both schools serve pizza again?

Solution: The solution to this problem will be the least common multiple (LCM) of 15 and 20. Students should be able to explain that the least common multiple is the smallest number that is a multiple of 15 and a multiple of 20.
One way to find the least common multiple is to find the prime factorization of each number: 2^2 x 5 = 20 and 3 x 5 = 15. To be a multiple of 20, a number must have 2 factors of 2 and one factor of 5 (2 x 2 x 5). To be a multiple of 15, a number must have factors of 3 and 5. The least common multiple of 20 and 15 must have 2 factors of 2, one factor of 3, and one factor of 5 (2 x 2 x 3 x 5) or 60.

| Cluster	
Apply and extend previous understandings of numbers to the system of rational numbers.	
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, origin, quadrants, coordinate plane, ordered pairs, *x*-axis, *y*-axis, coordinates.	
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**6.NS.5** Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. Example 1: a. Use an integer to represent 25 feet below sea level b. Use an integer to represent 25 feet above sea level. c. What would 0 (zero) represent in the scenario above? *Solution:* a. -25 b. +25 c. 0 would represent sea level.
6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	**6.NS.6** In elementary school, students worked with positive fractions, decimals and whole numbers on the number line and in quadrant 1 of the coordinate plane. In 6th grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e., thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistant from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as "the opposite of 4" which would be negative 4. In the example, – (–6.4) would be read as "the opposite of the opposite of 6.4" which would be 6.4. Zero is its own opposite.
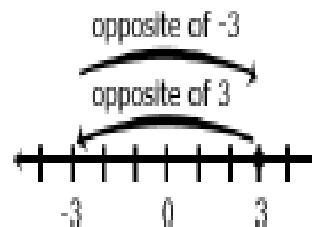
a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.	Example 1: What is the opposite of $2\frac{1}{2}$? Explain your answer. *Solution:* $-2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	Students worked with Quadrant I in 5th Grade. As the x-axis and y-axis are extended to include negatives, students begin to with the Cartesian Coordinate system. Students recognize the point where the x-axis and y-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (–, +). Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change in the x-coordinates from (-2, 4) to (2, 4), represents a reflection across the y-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	Example 1: Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the x-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point? $$\left(\frac{1}{2}, -3\frac{1}{2}\right) \qquad \left(-\frac{1}{2}, -3\right) \qquad (0.25, 0.75)$$ *Solution:* The coordinates of the reflected points would be $(\frac{1}{2}, 3\frac{1}{2})$, $(-\frac{1}{2}, 3)$, $(0.25, -0.75)$. Note that the y-coordinates are opposites. Example 2: Students place the following numbers would be on a number line: $-4.5, 3.2, -3\frac{3}{5}, -2, 0.2, 2, \frac{11}{2}$. Based on number line placement, numbers can be placed in order. *Solution:* The numbers in order from least to greatest are: $-4.5, -3\frac{3}{5}, -2, 0.2, 2, 3.2, \frac{11}{2}$ Students place each of these numbers on a number line to justify this order.

6.NS.7	**6.NS.7** Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line.
Understand ordering and absolute value of rational numbers.	

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6.NS.7

Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line.
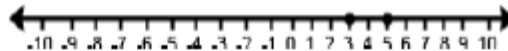
For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.

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6.NS.7 Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line.

Common models to represent and compare integers include number line models, temperature models, and the profit loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. ***Operations with integers are not the expectation at this level.***
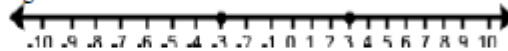
In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.
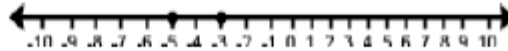
Case 1: Two positive numbers

$5 > 3$
5 is greater than 3
3 is less than 5

Case 2: One positive and one negative number

$3 > -3$
positive 3 is greater than negative 3
negative 3 is less than positive 3

Case 3: Two negative numbers

$-3 > -5$
negative 3 is greater than negative 5
negative 5 is less than negative 3

Example 1:

Write a statement to compare $-4\frac{1}{2}$ and –2. Explain your answer.

Solution:

$-4\frac{1}{2} < -2$ because $-4\frac{1}{2}$ is located to the left of –2 on the number line.

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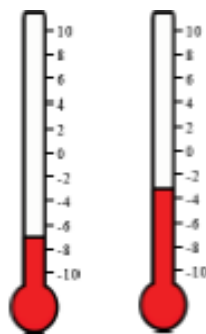|	
	Students recognize the distance from zero as the absolute value or magnitude of a rational number. **Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order.**
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.	Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than."
example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C.	
	Example 1: The balance in Sue's checkbook was –$12.55. The balance in John's checkbook was –$10.45. Write an inequality to show the relationship between these amounts. Who owes more?
	Solution: –12.55 < –10.45; Sue owes more than John. The interpretation could also be "John owes less than Sue."
	Example 2: One of the thermometers shows -3°C and the other shows -7°C. Which thermometer shows which temperature? Which is the colder temperature? How much colder? Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.
	Solution: • The thermometer on the left is -7 degrees; right is -3 degrees • The left thermometer is colder by 4 degrees • Either -7 < -3 or -3 > -7

	Although 6.NS.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context. Example 3: A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature: Albany 5° Anchorage -6° Buffalo -7° Juneau -9° Reno 12° *Solution:* Juneau -9° Buffalo -7° Anchorage -6° Albany 5° Reno 12°
c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*	Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value. Example 1: Which numbers have an absolute value of 7? *Solution:* 7 and –7 since both numbers have a distance of 7 units from 0 on the number line. Example 2: What is $\|-3\frac{1}{2}\|$? *Solution:* $3\frac{1}{2}$ In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write $\|-900\| = 900$ to describe the distance below sea level. When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases.

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	For example, –24 is less than –14 because –24 is located to the left of –14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of –24 is greater than the absolute value of –14. For negative numbers, as the absolute value increases, the value of the negative number decreases. Students find the distance between points when ordered pairs have the same x-coordinate (vertical) or same y-coordinate (horizontal). Example 1: What is the distance between (–5, 2) and (–9, 2)? *Solution:* The distance would be 4 units. This would be a horizontal line since the *y*-coordinates are the same. In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. ($\mid 9 \mid - \mid 5 \mid$). Coordinates could also be in two quadrants and include rational numbers. Example 2: What is the distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$? *Solution:* The distance between $(3, -5\frac{1}{2})$ and $(3, 2\frac{1}{4})$ would be $7\frac{3}{4}$ units. This would be a vertical line since the x coordinates are the same. The distance can be found by using a number line to count from $-5\frac{1}{2}$ to $2\frac{1}{4}$ or by recognizing that the distance (absolute value) from $-5\frac{1}{2}$ to 0 is $5\frac{1}{2}$ units and the distance (absolute value) from 0 to $2\frac{1}{4}$ is $2\frac{1}{4}$ units so the total distance would be $5\frac{1}{2} + 2\frac{1}{4}$ or $7\frac{3}{4}$ units. Students may also graph coordinates for polygons and find missing vertices based on properties of triangles and quadrilaterals.

6.NS.9
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0.

*For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

Students add and subtract rational numbers. Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with these operations. The expectation of the CCSS is to build on student understanding of number lines as it develops in 6th grade. Also, Algebra Tiles and Two-Colored Counters can be powerful manipulatives that can aid students in creating concrete models as they begin to discover and make sense of adding and subtracting integers as well as reinforce the concept of "opposite". This can be beneficial, as it will lay the foundation for developing concrete representations for solving equations by inverse operations.
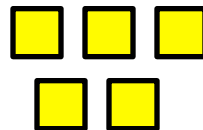
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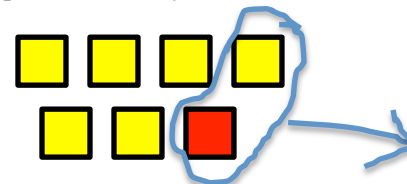 Red Tile = - 1
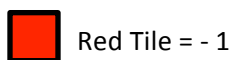
 Yellow Tile = 1

Example 1:
Teacher: Use your Algebra Tiles to model + 5 and using the key, draw a picture of the model you create.
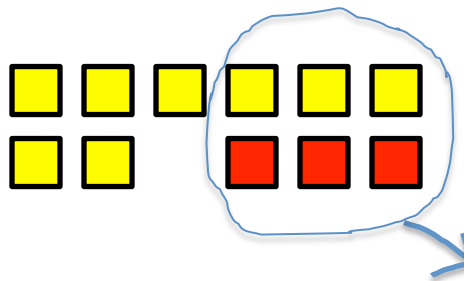
Student:

Teacher: Draw two more models that represent +5.

Student 1: My model represents + 5 because I inserted a *zero pair,* an extra yellow tile and a red tile (its opposite) to represent zero. So my model represents +5.

Student 2: My model represents + 5 because I inserted 3 *zero pairs,* 3 extra yellow tiles and 3 red tiles (an opposite for each extra yellow tile) to represent 3 sets of zero. So my model represents +5
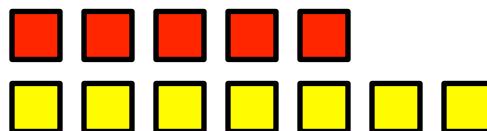
Example 2:

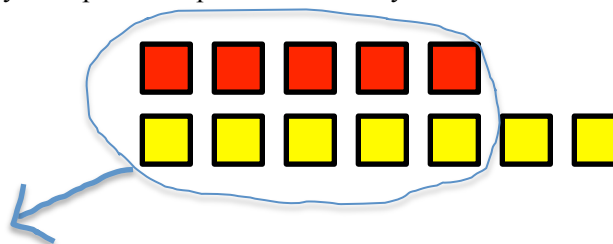Teacher: Use the Algebra Tiles to solve the following problem. Draw a picture of your work.

-5 + 7

Teacher: Tell me about how you solved this problem.

Student: First I modeled the problem by laying out 5 red tiles to represent -5 and then 7 yellow tiles to represent the action of adding +7 to my model.

Next, I grouped up my zero pairs and pulled them away.

This left me with my solution of +2.

Student Cont.: Even though this is an addition problem, I'm starting to notice that when I add a positive integer to a negative integer, I am "taking away" zero pairs. Before I found my solution of +2, I already knew my answer would be positive because I could clearly see that I had more yellow tiles than red tiles.
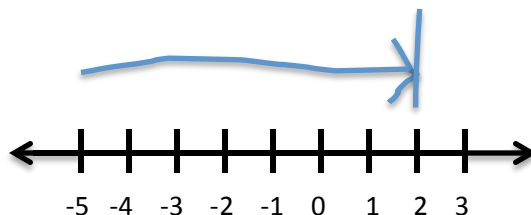
Teacher: That is a great observation! Has anyone else noticed this? Would anyone else like to share?
Students begin to discover and make sense the rules for the integer operations of addition and subtraction vs. being given meaningless rules and being asked to memorize them

Example 1:
Teacher:

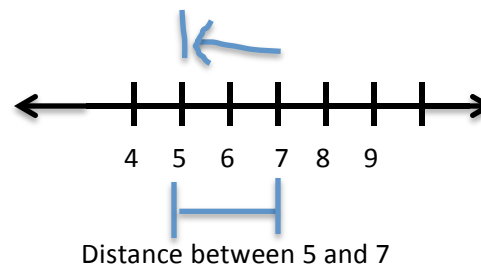Use a number line to add -5 + 7.

-5 -4 -3 -2 -1 0 1 2 3

Student:
I started at -5 on the number line and moved in the positive direction (to the right) 7 since I was adding 7.

The stopping point of 2 is the sum of this expression. Students also add negative fractions and decimals and interpret solutions in given contexts.

In 6th grade, students find the distance of horizontal and vertical segments on the coordinate plane. Students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line. In the example, 7 – 5, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). Therefore the answer would be 2.

4 5 6 7 8 9

Distance between 5 and 7

Example 2:
Use a number line to subtract: -6 – (-4)

Solution:
This problem is asking for the distance between -6 and -4. The distance between -6 and -4 is 2 and the direction from -4 to -6 is left or negative. The answer would be -2.

Example 3:
Use a number line to illustrate:
- $p - q$ ie. $7 - 4$
- $p + (-q)$ ie. $7 + (-4)$
- Is this equation true $p - q = p + (-q)$?

Students explore the above relationship when p is negative and q is positive and when both p and q are negative. Is this relationship always true?

Example 4:
Morgan has $4 and she needs to pay a friend $3. How much will Morgan have after paying her friend?

Solution:
$4 + (-3) = 1$ or $(-3) + 4 = 1$

Expression and Equations	
Cluster	
Apply and extend previous understanding of arithmetic to algebraic expressions.	

Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: exponents, base, numerical expressions, algebraic expressions, evaluate, sum, term, product, factor, quantity, quotient, coefficient, constant, like terms, equivalent expressions, variables.

| **6.EE.1**
Write and evaluate numerical expressions involving whole-number exponents. | Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $\left(\frac{1}{2}\right)^5$ can be written $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ which has the same value as $\frac{1}{32}$). Students recognize that an expression with a variable represents the same mathematics (i.e. x^5 can be written as $x \cdot x \cdot x \cdot x \cdot x$) and write algebraic expressions from verbal expressions.

Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations with exponents is the focus in 6th grade. |

Example 1:
What is the value of:

- 0.2^3 *Solution:* 0.008
- $7^2 - 24 \div 3 + 26$ *Solution:* 67

Example 2:
What is the area of a square with a side length of $3x$?   *Solution:* $3x \cdot 3x = 9x^2$

Example 3:
$4^x = 64$ *Solution:* $x = 3$ because $4 \cdot 4 \cdot 4 = 64$

Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, n" could be represented with $5n$ and that a number and letter written together means to multiply. All rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. **It is *important* for students to read algebraic expressions in a manner that reinforces that a variable represents a number**.

Example Set 1:
Students read algebraic expressions:

- $r + 21$ as "some number plus 21" as well as "r plus 21"
- $n \times 6$ as "some number times 6" as well as "n times 6"
- and $s \div 6$ as "as some number divided by 6" as well as "s divided by 6"

Example Set 2:
Students write algebraic expressions:

- 7 less than 3 times a number   *Solution:* $3x - 7$
- 3 times the sum of a number and 5   *Solution:* $3 (x + 5)$
- 7 less than the product of 2 and a number   *Solution:* $2x - 7$
- Twice the difference between a number and 5   *Solution:* $2(z - 5)$
- The quotient of the sum of x plus 4 and 2   *Solution:* $\frac{x+4}{2}$

6.EE.2
Write, read, and evaluate expressions in which variables stand for numbers.

a. Write expressions that record operations with numbers and with variables standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 - y$.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.

For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms.
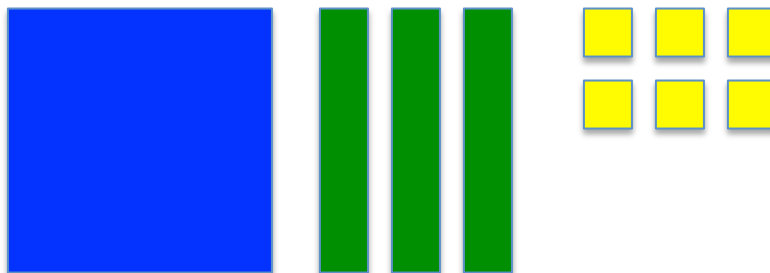
Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent.

Modeling expressions with Algebra Tiles can be powerful for students as they learn to identify parts of an expression but can also serve as a concrete model for simplifying expressions, helping students to distinguish between like and unlike terms. See introductory activities and tasks for Algebra Tiles:

Consider the following expression:
$x^2 + 3x + 6$

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole- number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = ½.

The variable is x.
There are 3 terms, x^2, $3x$, and 6.
There are 2 variable terms, x^2 and $3x$. They have coefficients of 1 and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term $3x$ represents $3x$'s or $3 \cdot x$. There is one constant term, 6. The expression represents a sum of all three terms.

Order of operations is introduced throughout elementary grades, including the use of grouping symbols, ( ), { }, and [ ] in 5th grade. Order of operations *with exponents* is the focus in 6th grade.

See article below for good instructional tasks that support this Standard.
Ameis, J. A. (2011). The truth about PEDMAS. Mathematics Teaching in the Middle School, 16(7), 414-420.

Students evaluate algebraic expressions, using order of operations as needed. Problems such as Example 1 below require students to understand that multiplication is intended when numbers and variables are written together and to use the order of operations to evaluate.

Example 1:
Evaluate the expression $2x + 2$ when x is equal to 4.

Student:
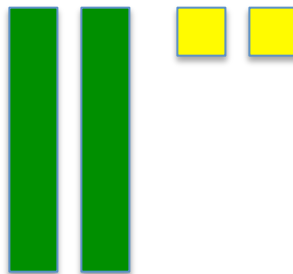
I represented the expression with Algebra Tiles. Then I substituted each x value with 4 since that was the given quantity for x. Now I can see that I have 2 groups of 4.

Then I was able to write a mathematical sentence and solve the problem.

$$2x + 2 = 2(4) + 2$$
$$= 8 + 2$$
$$= 10$$

Example 2:
Evaluate $5(n + 3) - 7n$, when $n = \frac{1}{2}$.

Student: I remembered from working with Algebra Tiles that I would replace the n values with $\frac{1}{2}$ since that is the given quantity.
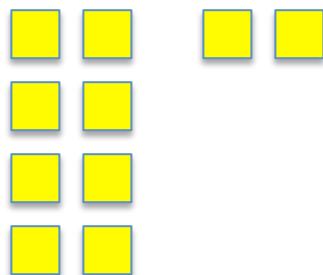
$5(\frac{1}{2} + 3) - 7(\frac{1}{2})$

Student: I see that I have 5 groups of $3\frac{1}{2}$, and I am taking away 1group of $3\frac{1}{2}$. So now I have 4 groups of $3\frac{1}{2}$. Multiply 4 times $3\frac{1}{2}$ to get 14.

$5(\frac{1}{2} + 3) - 7(\frac{1}{2})$

$5 (3\frac{1}{2}) - 3\frac{1}{2}$ Note: $7(\frac{1}{2}) = \frac{7}{2} = 3\frac{1}{2}$

$17\frac{1}{2} - 3\frac{1}{2}$

14

Example 3:
Evaluate $7xy$ when $x = 2.5$ and $y = 9$.

Solution: Students recognize that two or more terms written together indicate multiplication.
7 (2.5) (9)
157.5

|	
6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*	In 5th grade students worked with the grouping symbols ( ), [ ], and { }. Students understand that the fraction bar can also serve as a grouping symbol (treats numerator operations as one group and denominator operations as another group) as well as a division symbol. Example 4: Evaluate the following expression when $x = 4$ and $y = 2$ $$\frac{x^2 + y^3}{3}$$ *Solution:* $$\frac{(4)^2 + (2)^3}{3}$$ *substitute the values for x and y* $$\frac{16 + 8}{3}$$ *raise the numbers to the powers* $$\frac{24}{3}$$ *divide 24 by 3* 8 Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. Example 5: It costs \$100 to rent the skating rink plus \$5 per person. Write an expression to find the cost for any number (*n*) of people. What is the cost for 25 people? *Solution:* The cost for any number (*n*) of people could be found by the expression, $100 + 5n$. To find the cost of 25 people substitute 25 in for *n* and solve to get $100 + 5 \times 25 = 225$.

6.EE.4

Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary, students illustrate the distributive property with variables. Properties are introduced throughout elementary grades (3.OA.5); however, there has not been an emphasis on recognizing and naming the property. In 6th grade students are able to use the properties and identify by name as used when justifying solution methods (see example 4).

Example 1:
Given that the width is 4.5 units and the length can be represented by x + 3, the area of the flowers below can be expressed as $4.5(x + 3)$ or $4.5x + 13.5$.
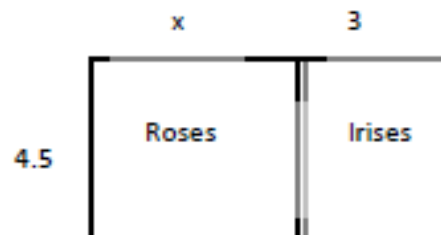
When given an expression representing area, students need to find the factors.

Example 2:
The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3).
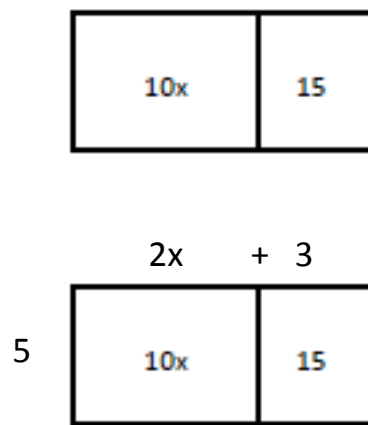
Example 3:
Students use their understanding of multiplication to interpret *3 (2 + x) as 3 groups of (2 + x)*. They use a model to represent x, and make an array to show the meaning of *3(2 + x)*. They can explain why it makes sense that *3(2 + x)* is equal to *6 + 3x*.

Student: At my elementary school, we learned that 3 x 2 means "3 groups of 2." So, I think of 3 (2 + x) as "3 groups of (2 + x)." I can build that with my Algebra Tiles and make 3 rows with 2 + x in each row.
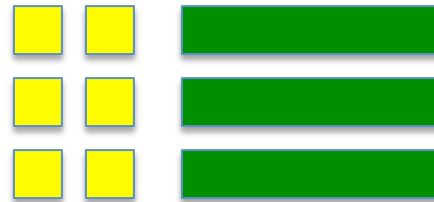
Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, 3x + 4x are like terms and can be combined as 7x; however, $3x + 4x^2$ are not like terms since the exponents with the *x* are not the same. Representing these expressions with Algebra Tiles provides students with a concrete model as the make sense of this abstract concept.

Student: When I represent 3x + 4x, I can see that all together I have 7x.
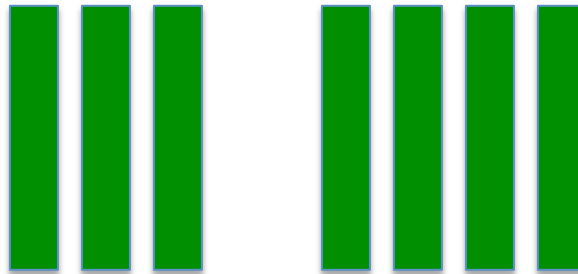
<u>Student:</u> When I represent $3x + 4x^2$, I can see that unless I know the value of x I cannot combine these terms because they are not alike.

This concept can be illustrated by substituting in a value for x. For example, $9x - 3x = 6x$ not 6. Choosing a value for x, such as 2, can prove non-equivalence.

$$9(2) - 3(2) = 6(2) \qquad \text{however} \qquad 9(2) - 3(2) = 6$$
$$18 - 6 = 12 \qquad\qquad\qquad\qquad 18 - 6 = 6$$
$$12 = 12 \qquad\qquad\qquad\qquad 12 \neq 6$$

Students can also generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that expressions are equivalent by simplifying each expression into the same form.

Example 1:
Are the expressions equivalent? Explain your answer.
4m + 8, 4(m+2), 3m + 8 + m, 2 + 2m + m + 6 + m

Solution:

Expression	Simplifying the Expression	Explanation
4m + 8	4m + 8	Already in simplest form
4(m+2)	4(m+2) 4m + 8	Distributive property
3m + 8 + m	3m + 8 + m 3m + m + 8 4m + 8	Combined like terms
2 + 2m + m + 6 + m	2m +m +m +2 + 6 4m + 8	Combined like terms Combined like terms

Cluster	
Reason about and solve one-variable equations and inequalities.	
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: inequalities, equations, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤, profit, exceed.	
6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	In elementary grades, students explored the concept of equality. In 6th grade, students explore equations as expressions being set equal to a specific value. The solution is the value of the variable that will make the equation or inequality true. Students use various processes to identify the value(s) that when substituted for the variable will make the equation true. Example 1: Joey had 26 papers in his desk. His teacher gave him some more, and now he has 100. How many papers did his teacher give him? This situation can be represented by the equation $26 + n = 100$ where n is the number of papers the teacher gives to Joey. This equation can be stated as "some number was added to 26 and the result was 100." Students ask themselves, "What number was added to 26 to get 100?" to help them determine the value of the variable that makes the equation true. Students could use several different strategies to find a solution to the problem: Student 1 (adding up): 26 + 70 is 96 and 96 + 4 is 100, so the number added to 26 to get 100 is 74. Student 2: (backing down) I know that 100 – 20 is 80 and I know that 80 – 6 is 74, so 100 – 26 is 74. Example 2: The equation $0.44\,s = 11$ where s represents the number of stamps in a booklet. The booklet of stamps costs 11 dollars, and each stamp costs 44 cents. How many stamps are in the booklet? Explain the strategies used to determine the answer. Show that the solution is correct using substitution. *Solution:* I tried to estimate to start. 0.44 x 10 would be 4.40. Double that – 0.44 x 20 – would be 8.80. So, the answer should be a little more than 20. I got my answer by dividing 11 (the cost of the booklet) by 0.44 (the cost of a stamp) to determine how many groups of 0.44 were in 11. I got 25, which works with my estimate. So, there are 25 stamps in the booklet.

6.EE.6
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
	Example 3:
	
	Twelve is less than 3 times another number can be shown by the inequality $12 < 3n$. What numbers could possibly make this a true statement?
	
	Solution:
	
	Since $3 \cdot 4$ is equal to 12 I know the value must be greater than 4. Any value greater than 4 will make the inequality true. Possibilities are 4.13, 6, $5\frac{3}{4}$, and 200. Given a set of values, students identify the values that make the inequality true.
	
	Students are not expected to solve inequalities formally (ex: stating the solution $n > 4$) at this stage.
	
	Students write expressions to represent various real-world situations.
	Example:
	   • Write an expression to represent Susan's age in three years, when a represents her present age.
	   • Write an expression to represent the number of wheels, w, on any number of bicycles.
	   • Write an expression to represent the value of any number of quarters, q.
	
	Solutions:
	   • $a + 3$
	   • $2w$
	   • $0.25q$
	
	Given a contextual situation, students define variables and write an expression to represent the situation.
	
	Example 2:
	The skating rink charges $100 to reserve the place and then $5 per person. Write an expression to represent the cost for any number of people.
	   n = the number of people
	   $100 + 5n$
	
	Students understand the inverse relationships that can exist between two variables. For example, if Sally has 3 times as many bracelets as Jane, then Jane has $\frac{1}{3}$ the amount of Sally. If s represents the number of bracelets Sally has, the $\frac{1}{3}s$ or $\frac{s}{3}$ represents the amount Jane has.

Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

More Examples:
- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.

  Solution: $2c + 3$ where *c* represents the number of crayons that Elizabeth has

- An amusement park charges $28 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.

  Solution: $28 + 0.35t$ where *t* represents the number of tickets purchased

- Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.

  Solution: $15h + 20 = 85$ where *h* is the number of hours worked

| **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form *x* +*p* = *q* and *px* = *q* for cases in which *p, q* and *x* are all nonnegative rational numbers. | Describe a problem situation that can be solved using the equation $2c + 3 = 15$; where *c* represents the cost of an item.

Possible solution:

Sarah spent $15 at a craft store.
• She bought one notebook for $3.
• She bought 2 paintbrushes for *c* dollars.
If each paintbrush cost the same amount, what was the cost of one brush? |

]

Students have used algebraic expressions to generate answers given values for the variable. This understanding is now expanded to equations where the value of the variable is unknown but the outcome is known.

For example, in the expression, $x + 4$, any value can be substituted for the x to generate a numerical answer; however, in the equation $x + 4 = 6$, there is only one value that can be used to get a 6. **Problems should be in context when possible and use only one variable.** Concrete models can also be helpful for students when solving equations and this is a great opportunity for students to look for an make use of structure, MP 7. Utilizing number puzzles can help students build number sense and flexibility. Number puzzles can also help students view the = sign as a sign of equivalence rather than an operational sign which is a common misconception.

Number Puzzle Example 1:

$3 + 4 = ? + 2$

Teacher: What number should replace the question mark?
Student 1: 7
Teacher: Will you explain how you arrived at this answer?
Student 1: Yes, 3 + 4 is 7

Teacher: Ok, I see what you are saying. Remember, an equal sign is making a statement that both sides of an equation are equivalent to one another. Why don't you find the sum of the left side of the equation and the sum of the right side of the equation to check your solution of 7. I'll come back to you so you can share what you find.

Student 2: I found a different solution.

Teacher: Please explain.

Student 2: Well, I re-wrote the left side of the equation so that it looked more like the right side.

$3 + 2 + 2 = ? + 2$

Student 2 cont.: I tried to make both sides of the equation look the same since they are equivalent to one another.

$5 + 2 = ? + 2$

Student 2 cont.: Now I can see for both sides to be equal, the ? would be replaced with a 5:   5 + 2 = 5 + 2

Example 2:

Teacher: Represent and solve $3x = 6$ with Algebra Tiles. Be sure to record your representation and how you solved the problem.

Student: First I represented the equation.

$$3x = 6$$
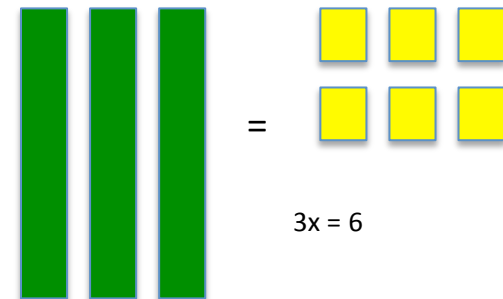
Student: I know that $3x = 6$ and I want to find the value of 1x. So I began to place a unit tile in each of the x tiles until there weren't any left.
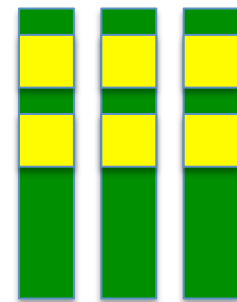
Student: I was able to place 2 unit tiles in each of my x tiles. This means that the value of each x tile must be 2. I can see that three groups of 2 are equal to 6.

Students write equations from real-world problems and then use inverse operations to solve one-step equations based on real world situations. Equations may include fractions and decimals with non-negative solutions.

Students recognize that dividing by 6 and multiplying by $\frac{1}{6}$ produces the same result. For example, $\frac{x}{6} = 9$ and $\frac{1}{6}x = 9$ will produce the same result.

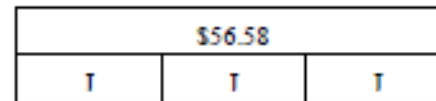Beginning experiences in solving equations require students to understand the meaning of the equation and the solution in the context of the problem.

Example 1:

Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

$56.58		
T	T	T

Sample Solution:
Student: "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled T is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3T =$ $56.58. To solve the problem, I need to divide the total cost of 56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10 x 3 is only 30 but less than $20 each because 20 x 3 is 60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That's $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 (15 + 3 + 0.86). I double check that the jeans cost $18.86 each because 3 x $18.86 is $56.58."

Example 2:
Julie gets paid $20 for babysitting. She spends $1.99 on a package of trading cards and $6.50 on lunch. Write and solve an equation to show how much money Julie has left.

20		
1.99	6.50	money left over (m)
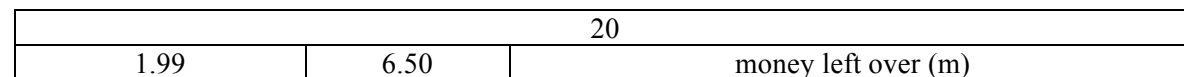
Solution: $20 = 1.99 + 6.50 + x$, $x = \$11.51$

Many real-world situations are represented by inequalities. Students write inequalities to represent real world and mathematical situations. Students use the number line to represent inequalities from various contextual and mathematical situations.

Example 1:

The class must raise at least $100 to go on the field trip. They have collected $20. Write an inequality to represent the amount of money, m, the class still needs to raise. Represent this inequality on a number line.

6.EE.8
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
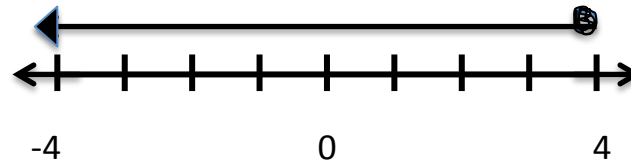
Solution:

The inequality $m \geq \$80$ represents this situation. Students recognize that possible values can include too many decimal values to name. Therefore, the values are represented on a number line by shading.

$70 $75 $80 $85 $90

A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the example above, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

Example 2:
Graph $x \leq 4$.

-4   0   4

Student: I know that x can be any value less than or equal to 4, so I made sure to color in my circle above the 4 since 4 is included in the solution set and shade my arrow to show that all values to the left of 4 are included.

Example 3:
The Flores family spent less than $200.00 last month on groceries. Write an inequality to represent this amount and graph this inequality on a number line.

Solution:
$200 < x$, where x is the amount spent on groceries.

Student: Since the groceries cost less than $200, I drew a line starting at $200. I made sure that I left an open circle above the 200 to show that this value is not included in my solution set. I stopped at 0 because you can't have negative money.

0   100   200
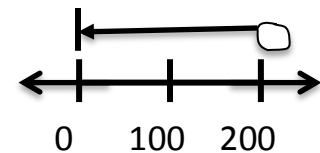
Cluster
Represent and analyze quantitative relationships between dependent and independent variables.
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: dependent variables, independent variables, discrete data, continuous data.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis. Students recognize that not all data should be graphed with a line. Data that is discrete would be graphed with coordinates only. Discrete data is data that would not be represented with fractional parts such as people, tents, records, etc. For example, a graph illustrating the cost per person would be graphed with points since part of a person would not be considered. A line is drawn when both variables can be represented with fractional parts. Students are expected to recognize and explain the impact on the dependent variable when the independent variable changes (As the x variable increases, how does the y variable change?) *Relationships should be proportional with the line passing through the origin.* Additionally, students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and/or a table of values. Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective. Example 1: What is the relationship between the two variables? Write an expression that illustrates the relationship. | x	1	2	3	4
y	2.5	5	7.5	10 | *Solution:* *y = 2.5x*

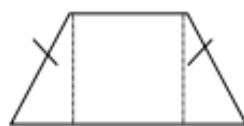| Geometry	
Cluster	
Solve real-world and mathematical problems involving area, surface area, and volume.	
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: area, surface area, volume, decomposing, edges, dimensions, net, vertices, face, base, height, trapezoid, isosceles, right triangle, quadrilateral, rectangles, squares, parallelograms, trapezoids, rhombi, kites, right rectangular prism.	
6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. "Knowing the formula" does not mean memorization of the formula. To "know" means to have an understanding of **why** the formula works and how the formula relates to the measure (area) and the figure. This understanding should be for *all* students. Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is ½ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is ½ bh or (b x h)/2. _TEACHER NOTE_: The language we use in discussing area at this stage is very important. It is common for teachers (and students) to say, "Area equals length times width" or "Area is base times height." But this is not always true. The areas of triangles, circles, trapezoids, (and other shapes) is not found by the formula $A = l \times w$. But once students "learn" that phrase, it is very difficult to get them away from it. It would be more accurate to say, "The area *of a rectangle* can be found by multiplying length times width." The following site helps students to discover the area formula of triangles. http://illuminations.nctm.org/LessonDetail.aspx?ID=L577 Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, and rhombi. Isosceles trapezoid Right trapezoid **Note:** Students recognize the marks on the isosceles trapezoid indicating the two sides have equal measure.

Example 1:
Find the area of the trapezoid shown below using the formulas for rectangles and triangles.
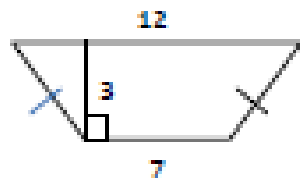
Solution:
The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2. The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there are a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5 units)(3 units) or 3.75 units^2.

Using this information, the area of the trapezoid would be:
21   units^2
  3.75 units^2
+3.75 units^2
28.5 units^2

Example 2:
A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

Solution:
The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches^2. The area of the new rectangle is 48 inches^2. The area increased 4 times (quadrupled). Students may also create a drawing to show this visually.

Example 3:
The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?
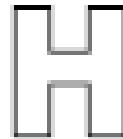
Solution:

Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches^2. The area of one index card is 24 inches^2. Divide 1728 inches^2 by 24 inches^2 to get the number of index cards. 72 index cards would be needed.

Example 4:

The sixth grade class at Hernandez School is building a giant wooden H for their school. The "H" will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.
1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?

Solution:
1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft^2. The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft. There are two of these pieces.
The area of the "H" would be 100 ft^2 – 18.75 ft^2 – 18.75 ft^2, which is 62.5ft^2.
A second solution would be to decompose the "H" into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore the area of the "H" would be 25 ft^2 + 25 ft^2 + 12.5 ft^2 or 62.5ft^2.
2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5 ft by 5 ft. Cut **two pieces of wood in half to create four pieces 5 ft. by 2.5 ft.** These pieces will make the two taller rectangles. A **third piece would be cut to measure 5 ft. by 2.5 ft.** to create the middle piece.

6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = B h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	Previously students calculated the volume of right rectangular prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula V = Bh (5.MD.3, 5.MD.4, 5.MD.5) The unit cube was 1 x 1 x 1. In 6th grade the unit cube will have fractional edge lengths. (i.e., ½ • ½ • ½ .) Students find the volume of the right rectangular prism with these unit cubes.

Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students *derive* the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

Example 1:

A right rectangular prism has edges of $1\frac{1}{4}$", 1" and $1\frac{1}{2}$". How many cubes with side lengths of ¼ " would be needed to fill the prism? What is the volume of the prism?

Solution:

The number of $\frac{1}{4}$" cubes can be found by recognizing the smaller cubes would be $\frac{1}{4}$" on all edges, changing the dimensions to $\frac{5}{4}$", $\frac{4}{4}$" and $\frac{6}{4}$". The number of one-fourth inch unit cubes making up the prism is 120 (5 x 4 x 6). Each smaller cube has a volume of $\frac{1}{64}$ ($\frac{1}{4}$" x $\frac{1}{4}$" x $\frac{1}{4}$"), meaning 64 small cubes would make up the unit cube.

Therefore, the volume is $\frac{5}{4}$x$\frac{6}{4}$x$\frac{4}{4}$ or$\frac{120}{64}$ (120 smaller cubes with volumes of$\frac{1}{64}$) or $1\frac{56}{64}$ →1 unit cube with 56 smaller cubes with a volume of $\frac{1}{64}$.

Example 2:
The model below shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ ft³.

$\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$

Example 3:

The model shows a rectangular prism with dimensions $\frac{3}{2}, \frac{5}{2}$, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{2}$ in. on each side. Students work with the model to illustrate $\frac{3}{2}$ x $\frac{5}{2}$ x $\frac{5}{2}$ = (3 x 5 x 5) x $\frac{1}{8}$. Students reason that a small cube has volume of $\frac{1}{8}$ in^3 because 8 of them fit in a unit cube.
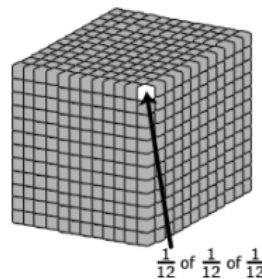
6.G.3

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
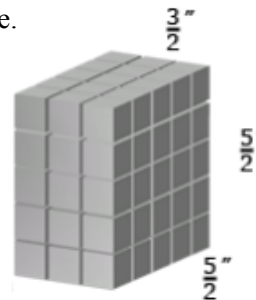
Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the *y*-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane.

Example 1:

If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.
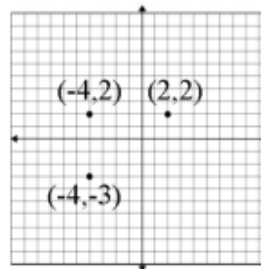
Solution:

To determine the distance along the *x*-axis between the point (-4, 2) and (2, 2) a student must recognize that -4 is |-4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |-4| + |2| . The length is 6 and the width is 5.

The fourth vertex would be (2, -3).

The area would be 5 x 6 or 30 units^2.

The perimeter would be 5 + 5 + 6 + 6 or 22 units.

Example 2:
On a map, the library is located at (-2, 2), the city hall building is located at (0, 2), and the high school is located at (0, 0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

Solution:

1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same *y*-coordinate. The distance between the *x*-coordinates is 2 (from -2 to 0).
2. The three locations form a right triangle. The area is 2 mi^2.

6.G.4
Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure.

Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205).

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Example 1:
Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

Example 2:
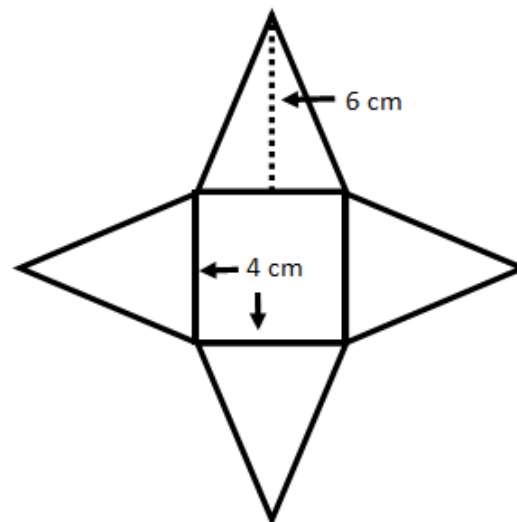Create the net for a given prism or pyramid, and then use the net to calculate the surface area.

Image taken from www.learnizillion.com .

| Statistics and Probability |
| Cluster |
| **Develop understanding of statistical variability.** |
| Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: statistics, data, variability, distribution, dot plot, histograms, box plots, median, mean. |

| **6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

 For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages. | Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e., documents).

 Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, "How tall am I?" is not a statistical question because there is only one response; however, the question, "How tall are the students in my class?" is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values. |

	Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: "How many hours per week on average do students at Jefferson Middle School exercise?"
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.	The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots.
	6-Trait Writing Rubric Scores for Organization
	Example 1: The dot plot to the right shows the writing scores for a group of students on organization. Describe the data.
	Solution: The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.473. If all students scored the same, the score would be approximately 3.50.
	NOTE: Mode as a measure of center and range as a measure of variability are not addressed in the standards and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (i.e., midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic.
	Example 1: Consider the data shown in the dot plot of the six trait scores for organization for a group of students.
	• How many students are represented in the data set?
	• What are the mean and median of the data set? What do these values mean? How do they compare?
	• What is the range of the data? What does this value mean?
	Solution:
	• 19 students are represented in the data set.
	• The mean of the data set is 3.5. The median is 3. The mean indicates that if the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.
	• The range of the data is 6, indicating the values vary 6 points between the lowest & highest scores.

Cluster
Summarize and describe distributions
Vocabulary: Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: box plots, dot plots, histograms, frequency tables, cluster, peak, gap, mean, median, interquartile range, measures of center, measures of variability, data, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier.

| **6.SP.4**
Display numerical data in plots on a number line, including dot plots, histograms, and box plots. | Students display data graphically using number lines. Dot plots, histograms, and box plots are three graphs to be used. Students are expected to determine the appropriate graph as well as read data from graphs generated by others.

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

A histogram shows the distribution of continuous data using intervals on the number line. The height of each bar represents the number of data values in that interval. In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students group the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the bin changes the appearance of the graph and the conclusions may vary from it.

A box plot shows the distribution of values in a data set by dividing the set into quartiles. It can be graphed either vertically or horizontally. The box plot is constructed from the five-number summary (minimum, lower quartile, median, upper quartile, and maximum). These values give a summary of the shape of a distribution. Students understand that the size of the box or whiskers represents the middle 50% of the data.

Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations: Box Plot Tool - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77 Histogram Tool -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Example 1:
Nineteen students completed a writing sample that was scored on organization. The scores for organization were 0,1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?
Solution:
 |

Example 2:

Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

Solution:

A histogram using 5 intervals (bins) (0-9, 10-19, …30-39) to organize the data is displayed below.

Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

Example 3:
Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

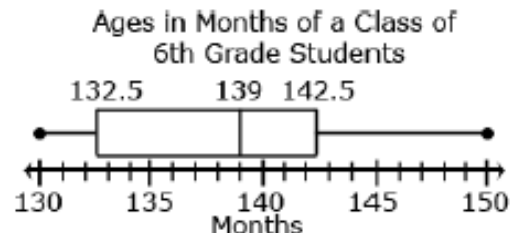| 130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Solution:
Five number summary
Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months

Ages in Months of a Class of
6th Grade Students

This box plot shows that
- $\frac{1}{4}$ of the students in the class are from 130 to 132.5 months old
- $\frac{1}{4}$ of the students in the class are from 142.5 months to 150 months old
- $\frac{1}{2}$ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

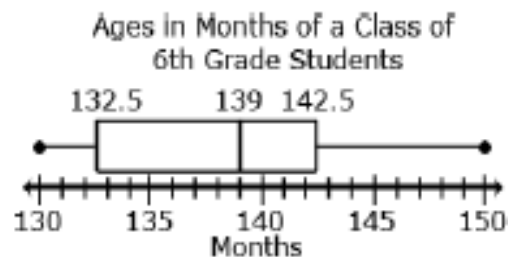|	
6.SP.5 Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range ~~and/or mean absolute deviation~~),as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), the number of observations, and summary statistics. Summary statistics include quantitative measures of center (median and median) and variability (interquartile range) including extreme values (minimum and maximum), mean, median, mode, range, and quartiles. Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable). <u>Measures of Center</u> Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it. The mean is the arithmetic average: the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point). Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean. Students find a missing value in a data set to produce a specific average. <u>Example 1:</u> Susan has four 20-point projects for math class. Susan's scores on the first 3 projects are shown below: Project 1: 18 Project 2: 15 Project 3: 16 Project 4: ?? What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning. *Solution:* One possible solution is to calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, so she needs to earn 19 points on Project 4 (68 – 49 = 19).

Measures of Variability

Measures of variability/variation can be described using the interquartile range.

The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

Example 1:
What is the IQR of the data below:

Ages in Months of a Class of
6th Grade Students

132.5   139 142.5

130   135   140   145   150
                 Months

Solution:
The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.

The interquartile range is represented by a single numerical value. Higher values represent a greater variability in the data.

Students understand how the measures of center and measures of variability are represented by graphical displays. Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

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